







SASPARM

Support Action for Strengthening Palestinian-administrated Areas capabilities for seismic Risk Mitigation

Call ID FP7-INCO.2011-6.2

MODULE 3: GROUND RESPONSE ANALYSES AND NEAR-SURFACE SITE CHARACTERIZATION

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SEISMIC PROSPECTING
USING SASW/MASW
TECHNIQUES
NNU, May 2 – 4, 2013













Why near-surface seismic?

Earthquake engineering - mainly 3 engineering parameters: Vs profile, Qs profile (damping ratio), and fundamental frequency

Exploration geophysics - 1.near-surface lithological properties (Vp/Vs ratio and Qp/Qs ratio linked to porosity, fluid saturation, mean grain size, fracture analysis) and 2. revoking near-surface disturbance effects (requirement for inverse filtering to improve imaging results of deeper strata)

Seismic imaging in general

Ex. Geomorphology, hydrogeology, archaeology, ore exploration, etc...







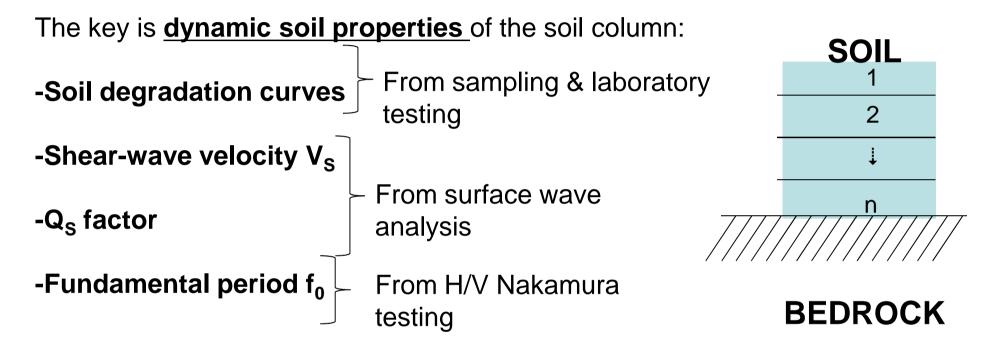








Quantifying site amplification



Why shear-waves?

Because S-waves are the most damaging during an earthquake!!













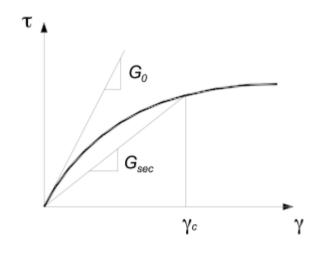
Elastic linear isotropic homogeneous medium:

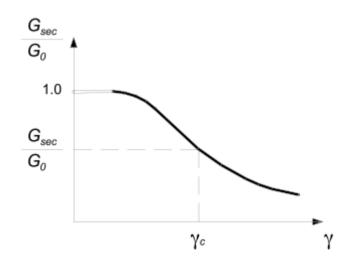
$$Vs = \sqrt{\frac{G}{\rho}}$$
 $G = \frac{\tau}{\gamma} = const$

Inelasticity: the shear stress-strain relationship

Initial shear modulus: $G_{
m max}=G_0$

For low shear strain amplitudes (<10-4%): $G_0 =
ho V s^2$

















Putting all information together from surface wave studies

2-station/multistation active/noise arrays:

Vs profile (and Qs profile)

Vs profile, we can also obtain Vs,30 and select soil classification for our site

$$V_{S,30} = \sum_{i=1,N} \frac{30}{\frac{d_i}{V_{Si}}}$$

EC8 Soil classification

Soil class	V _{s,30}
Α	> 800
В	360 - 800
С	180 - 360
D	< 180
E (C, D su A)	













Vs from Rayleigh waves













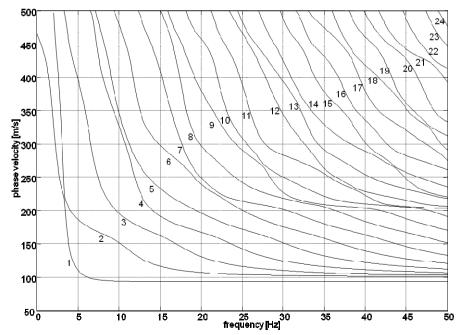
Vertically Inhomogeneous Elastic Media (surface waves)

-The solution to Love and Rayleigh wave eigenproblem is each not trivial.

-Each set
$$\left\{k_j, l_m^{(j)}\left(x_2, k_j, \omega\right)\right\}$$
 or $\left\{k_j, r_n^{(j)}\left(x_2, k_j, \omega\right)\right\}$ defines a mode of propagation

-In general there are: M normal modes at any given frequency.

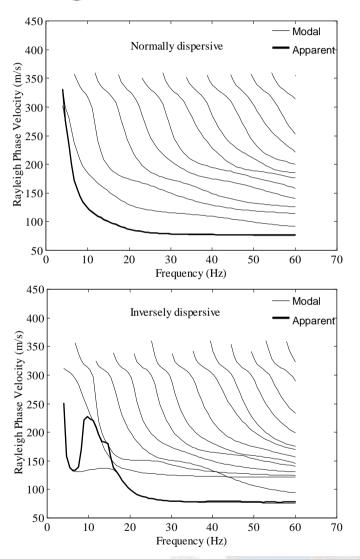
-If number of homogeneous layers overlaying homogeneous half-space is finite then the total number of surface wave modes of propagation is always finite (Ewing, 1957).







Higher Modes











Comparison between modal and apparent Rayleigh dispersion curves for an example of a normally dispersive (top) and inversely dispersive (bottom) half-space (from Tokimatsu, K. et al. (1992)).









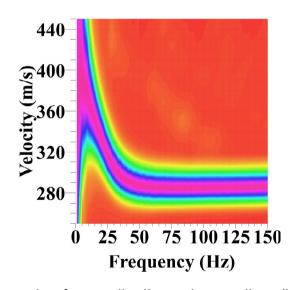


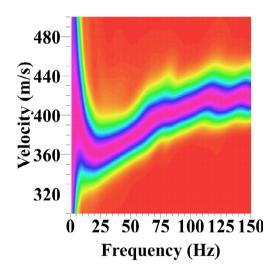


Higher Modes

Higher modes arise due to 1D heterogeneity.

Rule of thumb: fundamental mode is dominant if the medium is normally dispersive, otherwise higher modes may dominate.





Example of normally dispersive medium (left) and inversely dispersive medium (right). (Source Lai et al, 2013)













"For Rayleigh waves generated by harmonic sources, the various modes of propagation of surface waves are superimposed in a spatial Fourier series. The corresponding phase velocity of the Rayleigh waveform, which is the result of interference between the different modes, is termed in the literature apparent or effective Rayleigh wave phase velocity."

Source: Lai et al. 2013





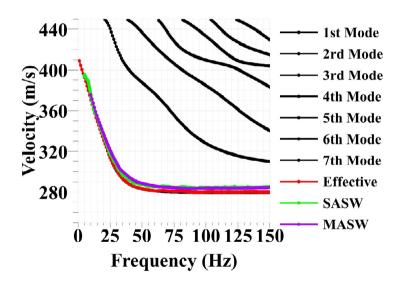


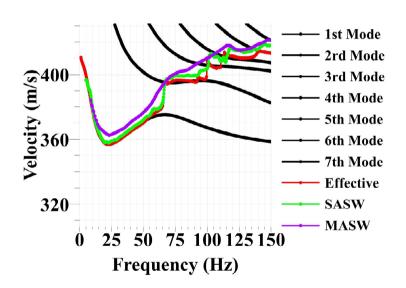






Summation of different mode contributions at each frequency give rise to the concept of 'effective' or 'apparent' phase velocity





Example of normally dispersive medium (left) and inversely dispersive medium (right). (Source Lai et al, 2013)



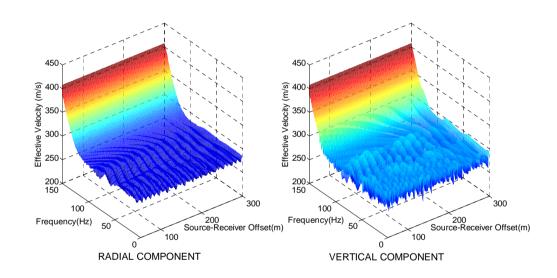












Example of dispersion surface showing dependence of effective Rayleigh wave phase velocity with frequency and distance. (Source: Lai et al, 2013)













$$\hat{V}_{r}(r,z,\omega) = \frac{2\omega \sum_{i=1}^{M} \sum_{j=1}^{M} \left\{ \frac{w_{1}(z,k_{i})w_{1}(z,k_{j})w_{2}(z_{s},k_{i})w_{2}(z_{s},k_{j})\cos\left[r(k_{i}-k_{j})\right]}{\left(V_{i}U_{i}I_{i}\right)\left(V_{j}U_{j}I_{j}\right)\sqrt{k_{i}k_{j}}} \right\}}{\frac{\sum_{i=1}^{M} \sum_{j=1}^{M} \left\{ \frac{w_{1}(z,k_{i})w_{1}(z,k_{j})w_{2}(z_{s},k_{i})w_{2}(z_{s},k_{j})(k_{i}+k_{j})\cos\left[r(k_{i}-k_{j})\right]}{\left(V_{i}U_{i}I_{i}\right)\left(V_{j}U_{j}I_{j}\right)\sqrt{k_{i}k_{j}}} \right\}}$$

$$\hat{V}_{z}(r,z,\omega) = \frac{2\omega \sum_{i=1}^{M} \sum_{j=1}^{M} \left\{ \frac{w_{2}(z,k_{i})w_{2}(z,k_{j})w_{2}(z_{s},k_{i})w_{2}(z_{s},k_{j})\cos\left[r(k_{i}-k_{j})\right]}{\left(V_{i}U_{i}I_{i}\right)\left(V_{j}U_{j}I_{j}\right)\sqrt{k_{i}k_{j}}} \right\}}{\sum_{i=1}^{M} \sum_{j=1}^{M} \left\{ \frac{w_{2}(z,k_{i})w_{2}(z,k_{j})w_{2}(z_{s},k_{i})w_{2}(z_{s},k_{j})(k_{i}+k_{j})\cos\left[r(k_{i}-k_{j})\right]}{\left(V_{i}U_{i}I_{i}\right)\left(V_{j}U_{j}I_{j}\right)\sqrt{k_{i}k_{j}}} \right\}}$$

Velocity components of the apparent Rayleigh wave phase velocity along directions r and z respectively. I stands for the Rayleigh wave energy integral, V and U are the phase and group velocities of each mode w are the eigenfunctions, k the wavenumber, r is the distance, z depth, ω frequency and z is depth.

Lai et al, 2013













Damping ratio from Rayleigh waves







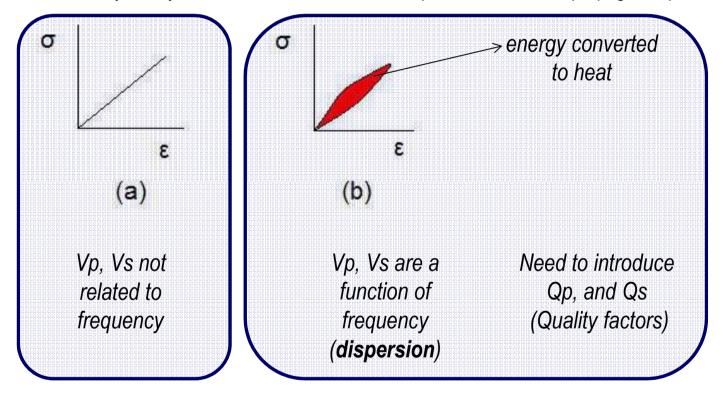






Elasticity vs. Viscoelasticity

Viscoelasticity theory is a consistent framework to explain seismic wave propagation phenomena during seismic testing.



Moreover, during seismic testing deformations are small hence linear viscoelasticity suffices!

Thus, we can study wave propagation using the elastic-viscoelastic correspondence theorem.





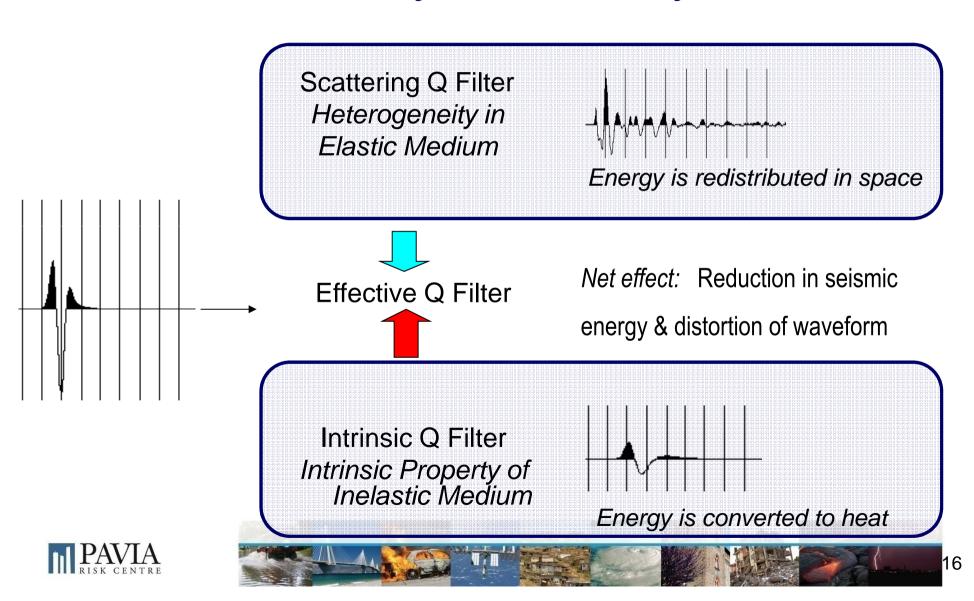








Linear Viscoelasticity: effect on body waves











Linear Viscoelasticity: effect on body waves

Mathematical expressions now contain complex bulk and shear moduli:

$$\mu \to \mu^*$$

$$K = \lambda + \frac{2}{3}\mu \to K^*$$

And consequently, the wavenumber (& velocities) are also complex. Alternatively, keep velocities real and introduce an attenuation term:

$$k_{\chi}^{*} = \frac{\omega}{V_{\chi}^{*}} = \left(\frac{\omega}{V_{\chi}} - i\alpha_{\chi}\right)$$

Moreover, velocities and attenuation are related to each other through Kramers-Krönig relations and the real and imaginary parts of the complex wavenumber are a Hilbert transform pair. This is the necessary and sufficient condition to satisfy principle of causality (?) (reaction can never precede the action).













Quality Factors (Damping ratio) vs. Attenuation

$$\frac{1}{Q} = \frac{\alpha c}{\pi f - \frac{\alpha^2 c^2}{4\pi f}}$$

Relationship between Q and attenuation

$$\frac{1}{Q} = \frac{\pi f}{c\alpha}$$

Relationship between Q and attenuation for low-loss medium

$$D = \frac{1}{2Q} = \frac{\alpha}{\omega}$$

Relationship between Q, attenuation and material damping ratio

$$c(\omega) = c(\omega_0) \left[1 - \frac{1}{\pi Q(\omega)} \ln \left(\frac{\omega_0}{\omega} \right) \right]$$
 Dispersion due to Q













Linear Viscoelasticity: effect on Rayleigh waves

Vertically inhomogeneous media: If weak dissipation is assumed, then the variational principle of Love and Rayleigh waves can be used to solve surface wave propagation in linear dissipative continua. Using variational calculus associated with weak dissipation assumption, we obtain:

$$\begin{cases}
V_R(\omega) = V_R^e + \left\{ \int_0^{\infty} V_S \left[\frac{\partial V_R}{\partial V_S} \right]_{\omega, V_P} \left[1 - \frac{V_S^e}{V_S} \right] dx_2 + \int_0^{\infty} V_P \left[\frac{\partial V_R}{\partial V_P} \right]_{\omega, V_S} \left[1 - \frac{V_P^e}{V_P} \right] dx_2 \right\}
\end{cases}$$

$$\left[\alpha_{R}\left(\omega\right) = \frac{\omega}{\left[V_{R}\left(\omega\right)\right]^{2}} \cdot \left\{\int_{0}^{\infty} V_{S} D_{S} \left[\frac{\partial V_{R}}{\partial V_{S}}\right]_{\omega, V_{P}} dx_{2} + \int_{0}^{\infty} V_{P} D_{P} \left[\frac{\partial V_{R}}{\partial V_{P}}\right]_{\omega, V_{S}} dx_{2}\right\}$$

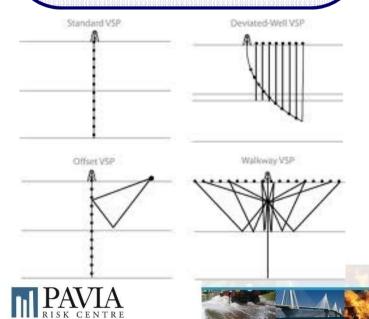




Seismic field testing

Invasive (downhole)

Cross-hole
Down-hole
Seismic Cone (SCPT)
Seismic Dilatometer (SDMT)
P-S Suspension Logging
Vertical Seismic Profiling (VSP)





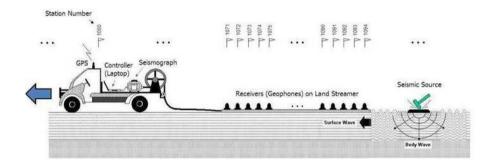






Non-invasive (surface)

Refraction
Reflection
Surface Wave Methods



Source: http://masw.com

http://www.co2crc.com.au/otway/mnv_seismic.html











Field Testing for Surface Waves





Seismic field testing









Invasive tests

- ++ 'Old' technology, well-documented and standardized
- ++ Excellent resolution to large depths
- ++ Direct measurements, hence interpretation is unique (usually)
- -- Relatively high cost
- -- Time consuming
- -- Invasive

Non-invasive tests

- ++ Large volume and averaging of properties
- ++ Cost efficient and time efficient
- ++ Great for 'sensitive' sites because non-invasive
- ++ Ambient noise testing particularly useful in urban noisy areas!!!
- -- Complex interpretation, processing/interpretation critical
- -- Resolution, especially at larger depths, less resolution
- -- Often nonuniqueness (especially for surface wave testing)













What are we trying to do?

Evidently, we want to observe the dispersive characteristics of the Rayleigh wave, so we need to have:

a seismic energy source (could be active or passive) and

An observation which is a seismic record (1 sensor, 2 sensors, more?). Recording is (most commonly) velocity time history r(t).

We will learn how to take r(t) and transform it mathematically in order to extract V_{Rayleigh}(f), which is termed the dispersion curve.

We will finally learn to take the dispersion curve, and invert for the shear-wave velocity profile.

Lastly, we will talk about Rayleigh wave attenuation curves, and inversion for the shear-wave velocity (Vs) and damping ratio (or Qs) profile.





Determining Vs









FORWARD PROBLEM Vertical particle motion Phase velocity V_R **Numerical** Wavelength $V_R = \lambda \cdot f$ V_{S2}> Frequency f **Dispersion Curve** Stiffness profile Long wavelength Short wavelength **Experimental** Low frequency High frequency **INVERSE PROBLEM**

Source: Foti, 2012





Determining Qs









FORWARD PROBLEM Vertical particle motion Attenuation ar Numerical Qs Z Frequency f **Attenuation Curve** Stiffness profile Long wavelength Short wavelength **Experimental** Low frequency High frequency **INVERSE PROBLEM**

Source: adapted from Foti, 2012





Assumptions









- 1. No lateral heterogeneity (in practice, some lateral heterogeneity is ok)
- 2. Only plane surface waves (body wave contribution is small)
- 3. Fundamental mode dominates

Advanced methods, use higher modes for inversion, or even the effective wave which is composed from a superposition of all the modes.



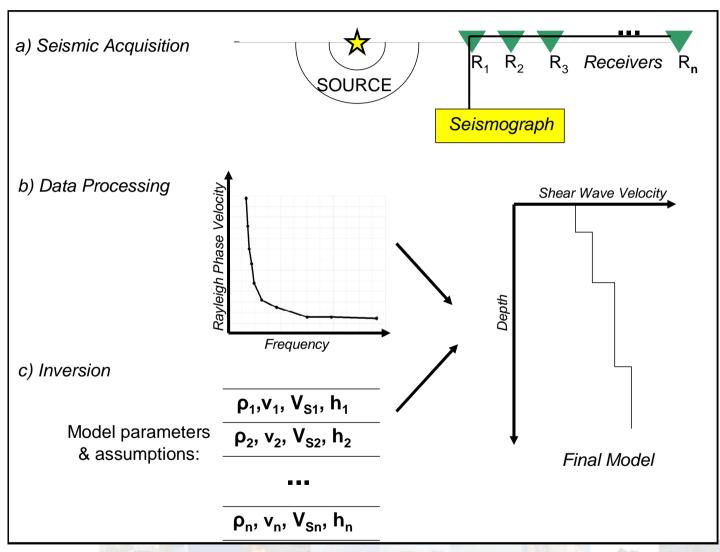








Schematic Representation of Vs inversion







Testing equipment





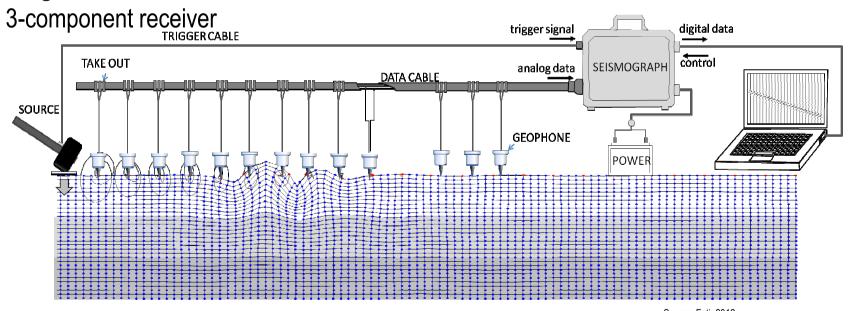




Multi-array methods:

Receivers,
Seismic spread cables,
Seismograph
Active source (trigger and trigger cable)

Single station H/V:







Receivers









Displacement, velocity-meters and accelerometers depending on whether they record particle displacement/velocity/acceleration
Also pressure-meters (hydrophones), however, no sense of polarity, mainly compression/dilation

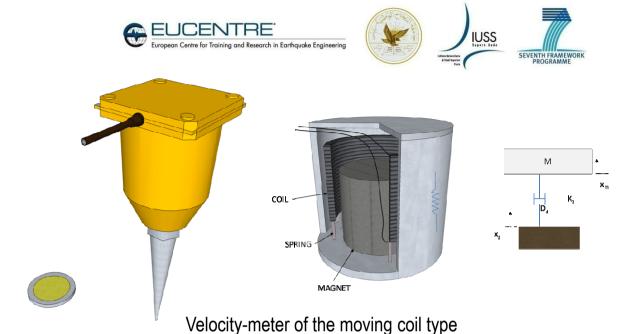
Most commonly for surface seismic analysis, **velocity-meters** are used because frequency range of interest is intermediate (displacement-meters usually used for strong-motion structure monitoring, and accelerometers for higher frequencies)

MEMS accelerometer: new technology for lower frequencies





Receivers



Two main dynamic parameters have to be considered to understand the effect of the receiver on the recorded signals: the natural frequency and the damping.

Natural frequency is the resonance frequency of the receiver, which controls the minimum usable frequency for the transducer.

The effect of the **damping** on the response curve is important as well, since the resonance peak at the natural frequency is flattened by the damping. It is recommended to set it to get a flat response in the frequency band of interest.





Active Seismic sources













Examples of impulsive sources (weight-drop, Betsy-gun and hammer)











Source: Ambient Noise









Noise recordings are used in single station (H/V) and multi-station (ReMi, SPAC, HRFK, seismic interferometry) seismic testing

But **WHAT** is noise, and **WHERE** does it come from?

Short history of noise

- -before 1950s: correlation between meteorological perturbations and microseisms
- -1950s to 1970s: beginning of seismic array analysis of noise thanks to the studies of Capon and Aki
- -since 1970s: the H/V method was developed and multistation array analysis for Vs inversion. The nature of noise has also been studied.





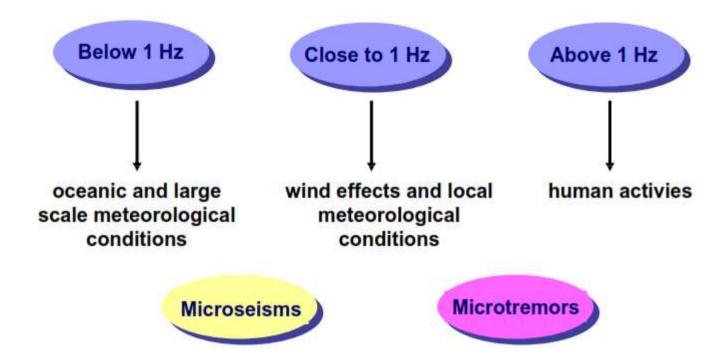
Origin of Ambient Noise























Linear Arrays





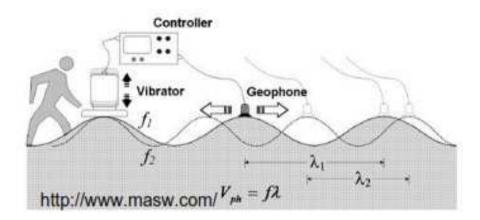








Steady-state method (Jones, 1958; 1962)



Vibrator excites a single frequency fi

Scan the ground with sensor. Locate maxima for each *f*_i

Evaluate distance xi between 2 consecutive maxima

Phase velocity is
$$c_i = x_i * f_i$$

Implicitly assumes only have a single mode













The 2-station method

In the two-station method, one employs two wave recordings, at distances x₁ and x₂ from their relative phase difference for each frequency, and given the distance, one can compute the Rayleigh phase velocity dependence on frequency (dispersion curve).

The two-station method was the basis for the development of the spectral-analysis-of-surface-waves (SASW) technique (Stokoe et al., 1994).

Still popular because uses a very limited equipment, which however has to be moved during the test in order to characterize different wavelengths.



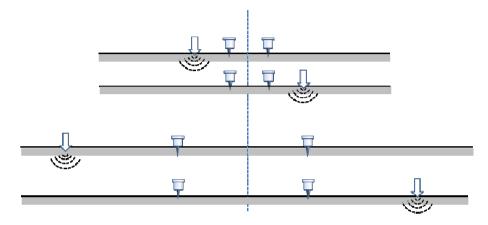












Two-station acquisition with the <u>common receiver midpoint scheme</u> with forward and reverse shots.

- Usually, Xsource-Xreceiver1 = Xreceiver1-Xreceiver2
- For a given spacing, only a limited range of frequencies can be obtained, due to spatial aliasing, attenuation, near field effects and so on.
- Short spacings and light sources used for high frequency (short wavelength), longer spacings and heavier sources used for the low frequency (long wavelength).



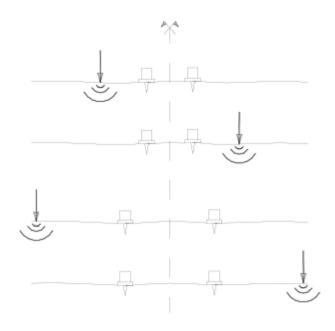












Common receiver midpoint array.



Common source array.



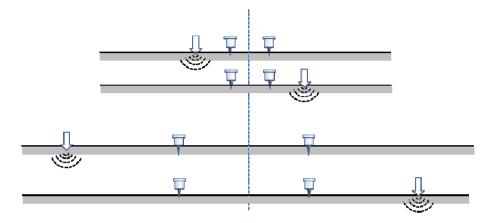












Two-station acquisition with the <u>common receiver midpoint scheme</u> with forward and reverse shots.

- Reversing the source location compensate for phase distortions of the receivers and can help in identifying the effect of coherent noise.
- The effect of multiple modes, lateral variations, and other coherent noise types is more critical with two receivers than in multichannel shot gathers.













Time domain signal recorded at each receiver:

$$s(x_m,t) m=1,2$$

Fourier transform is:

$$S(x_{m},\omega) = |S(x_{m},\omega)| e^{i[\phi(\omega) + k(\omega)x_{m}]}$$

Computing the cross-power spectrum:

$$S_{12}\left(\omega\right)^{i\left[\phi(\omega)+k(\omega)x_{2}\right]}=\left|S\left(x_{1},\omega\right)\right|\left|S\left(x_{2},\omega\right)\right|e^{ik(\omega)(x_{2}-x_{1})}$$

Taking the phase of the cross-power spectrum:

$$\theta_{12}(\omega) = \arg(\hat{S}_{12}(\omega))$$
$$= k(\omega)(x_2 - x_1)$$

Finally, obtain the Rayleigh wave phase velocity:

$$V_{R}(\omega) = \frac{\omega(x_{2} - x_{1})}{\theta_{12}(\omega)}$$

Important!! Need to unwrap phase:

$$\theta_{\text{unwrap}}(\omega) = \theta_{\text{wrap}}(\omega) \pm 2n\pi, \ n = 0, 1, 2...$$





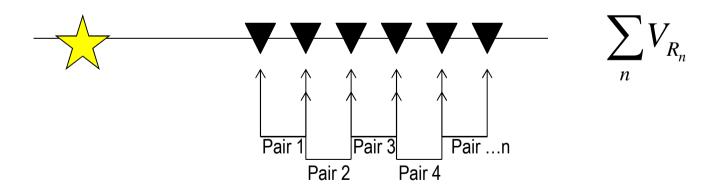








- Important!! If there are **higher modes** present (ex. Inversely dispersive medium), 2-station method yields the phase velocity of the **effective** Rayleigh wave (composed of superposition of higher modes!!).
- Effective Rayleigh wave properties vary with distance from source, hence in the 2-station method, some averaging (averaging over several receiver pairs at different offsets from the source) may be required...





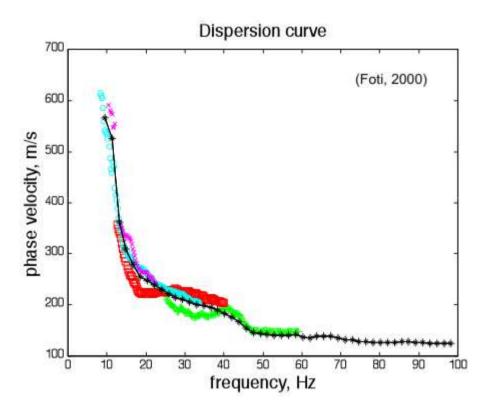












Averaging over different frequency bands.



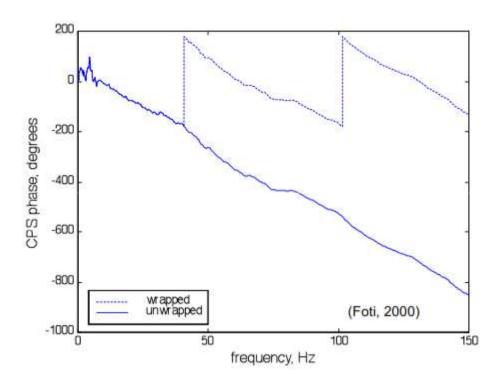












Phase unwrapping can be challenging in the presence of noise or a narrow frequency range



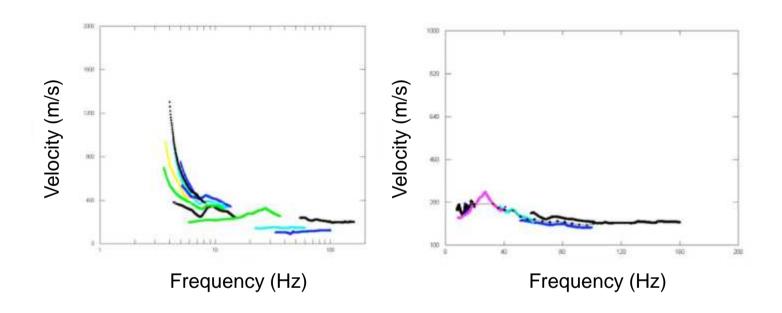












Example of problem with unwrapping: same dataset, different interpretation.













Multichannel Analysis of Surface Waves









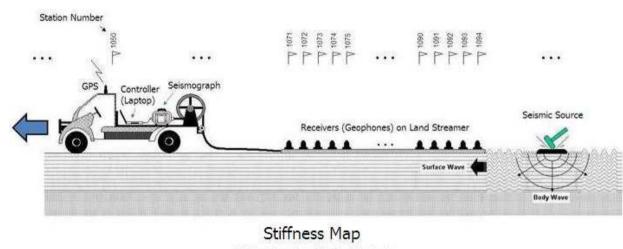


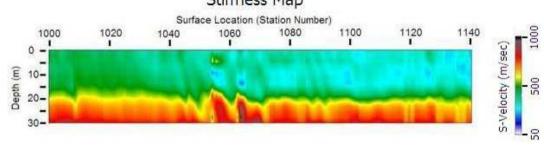


MASW

First studies: Al-Husseini et al., 1981; Mari, 1984; Gabriels et al., 1987

Later studies: Park et al., 1999; Foti, 2000









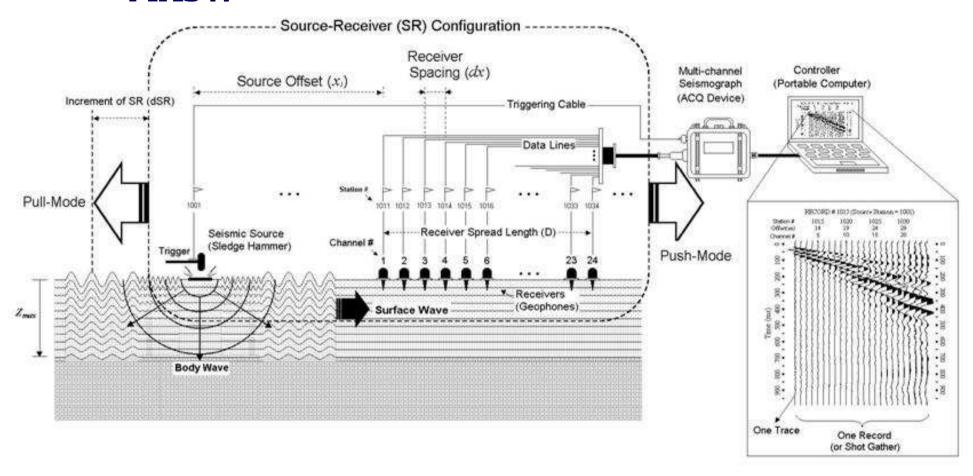








MASW







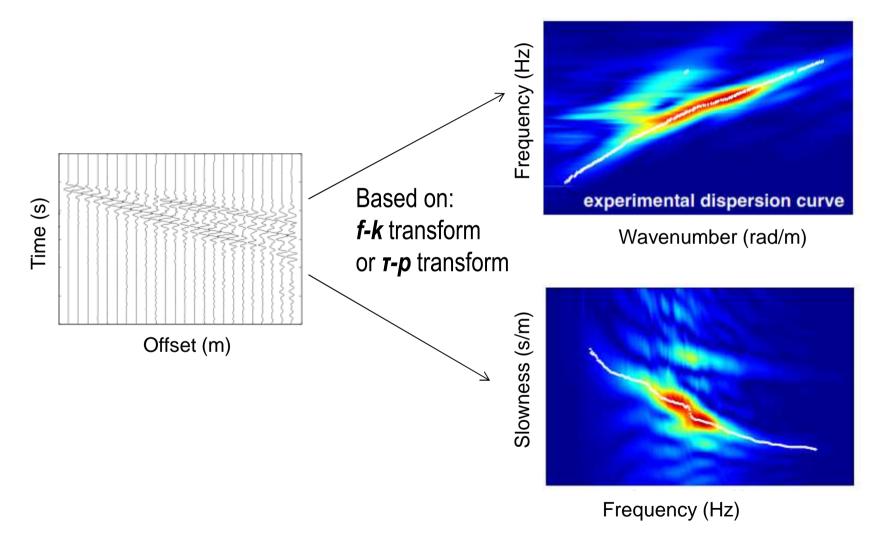








MASW







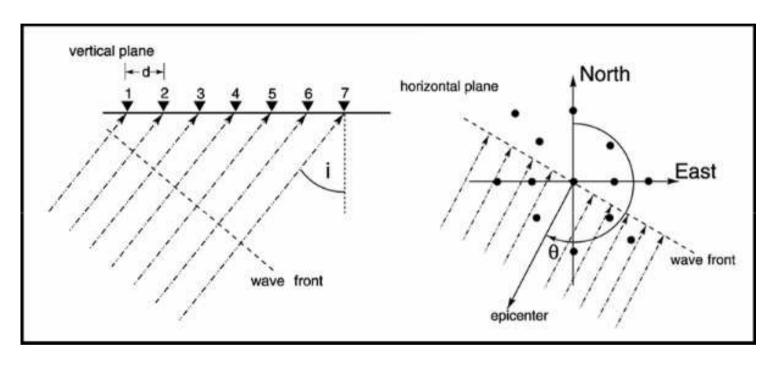








Transform-based methods: slowness



Geometry of plane wave: parameters of wave propagation





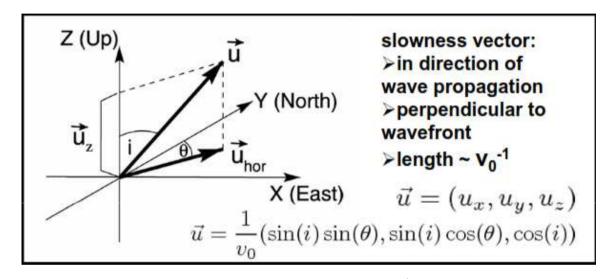








Transform-based methods: slowness



The slowness vector: $\overrightarrow{u} = u_{hor}(\sin(\theta), \cos(\theta), \frac{1}{\tan(i)})$

Horizontal slowness: $p = |\overrightarrow{u}_{hor}| = \frac{\sin i}{V_o}$

Source: Geopsy H/V tutorial seminar, Thessaloniki, 2010 organized by LGIT, ITSAK, Universitat Postdam, IRD













Transform-based methods: 7-p vs. f-k

т-р Method

$$s(x,t) \supset S(\omega,x)$$

$$U(\omega, p) = \int_{0}^{+\infty} x dx J_{o}(\omega p x) S(\omega, x)$$

$$U(\omega, p) \supset R(\tau, p)$$

1D Fourier Transform

Hankel Transform

1D Inverse Fourier Transform

f-k Method

$$G[k, f] = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{t=0}^{N-1} g(x, t) e^{-i2\pi \left(\frac{xk}{M} + \frac{tf}{N}\right)}$$

$$0 \le x, k \le M - 1, 0 \le t, f \le N - 1$$

2D Fourier Transform











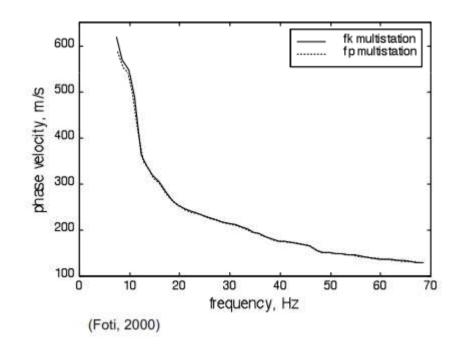


Transform-based methods: 7-p vs. f-k

The τ-p and frequency-wavenumber (or f-k) processes are closely related. Indeed, the τ-p process embodies the frequency-wavenumber transformation, so the use of this technique suffers the same limitations as the f-k technique. (Benoliel et al., 1987)

$$v_R(f) = \frac{1}{p\big|_{A=A_{\text{max}}}}$$

$$v_R(f) = \frac{2\pi \cdot f}{k\big|_{A=A_{\text{max}}}}$$







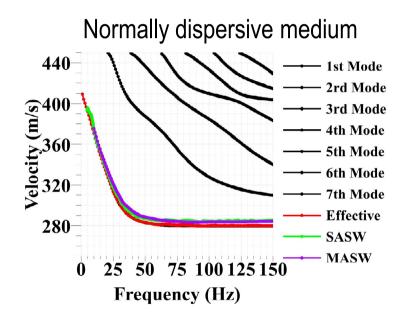




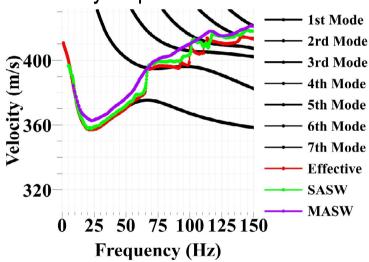




MASW vs. SASW







Good agreement between MASW and SASW, provided there is spatial averaging of SASW.

Source: Lai et al, 2013













MASW Pros and Cons

Pros: Simplicity

Identification of several modes

Cons: Near field effect

Influence of body waves (Tokimatsu, 1997; Sanchez-Salinero, 1987)

Cylindric propagation (Zywicki, 1999)

Far field effect

Attenuation of high frequencies

Low S/N due to intrinsic Q or scattering

Body head waves













ReMi Refraction Microtremor

- -Introduced by Louie (2001)
- -Noise recording using same configuration as MASW, but no active source!
- -Passive sources are natural and anthropogenic.
- -Suitable for urban areas where strong motion records are sparse, and where noise level is too high for good MASW results.
- -May have slightly different characteristics in dispersion curve compared to MASW due to differences in source.









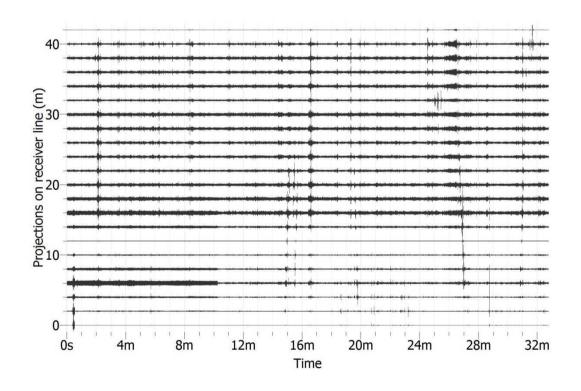




ReMi Refraction Microtremor

Acquisition:

For most V_{S,30} studies, a dt=2msec, and approximately 30 minutes of data is sufficient for good results.







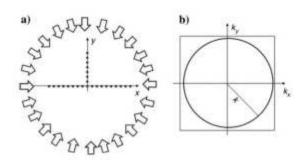


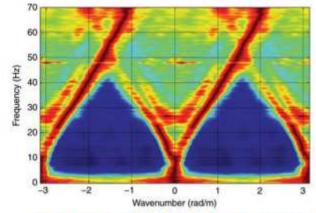




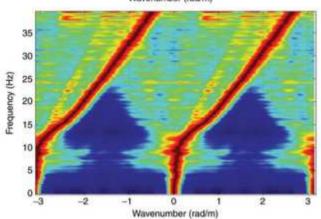
ReMi We need uniform noise distribution!

Uniform distribution of the source implies a symmetric fk spectrum





Example of non symmetric fk spectrum



(Strobbia & Cassiani, 2011)













ReMi vs. MASW

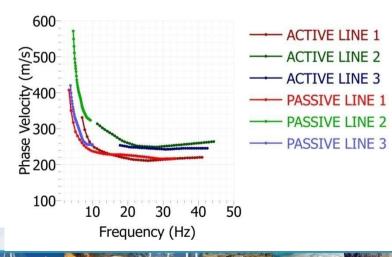
In their study Stephenson et al. (2005) conclude that neither of the two methods was consistently better

They suggest that their simultaneous use can be useful in urban settings.

Due to source spectra differences, ReMi may result in deeper imaging sometimes

Possibly construct a combined dispersion curve from MASW and Re.Mi.

data













Linear Array: finite and discrete

Limits due to sampling of seismic array

Array measurements can be seen as a discrete spatial sampling (receiver locations) of a continuous process (seismic wavefield)

For 1D linear arrays with equidistant spacing, the equivalence to time series sampling is straightforward:

Time Domain

$$\Delta T < T_{\min} / 2$$

$$\Delta\omega = 2\pi/((N-1)\Delta T)$$

Spatial Domain

$$\Delta x < \lambda_{\min}^* / 2$$

* apparent

$$\Delta k = 2\pi / ((N-1)d_{\min}) = 2\pi / D_{\max}$$













Linear Array: finite and discrete

Limits due to sampling of seismic array

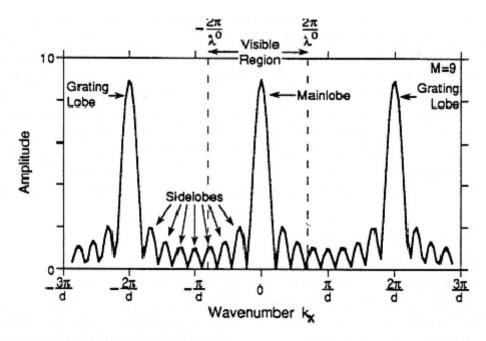


Figure 3.20 The aperture smoothing function magnitude |W(k)| for uniform shading is plotted for a nine-sensor regular linear array. This spatial spectrum has period $k=2\pi/d$. The visible region of the aperture smoothing function is that part for which $-2\pi/\lambda^o \le k_x^o \le 2\pi/\lambda^o$. What might be called secondary mainlobes—those not located at the origin—are termed grating lobes.















Recommendation on source-receiver distance

Normally dispersive media (body wave velocity increasing monotonically with depth) near field important up to $~\chi/2$

Inversely dispersive media near field may be important up to 2λ

Source: Holzlohner (1980), Vrettos (1991) and Tokimatsu (1995)

Recommended strategy: acquire data with different source-receiver offsets to recognize the near-field or use small offset and filter near-field during processing.

Socco and Strobbia (2004)







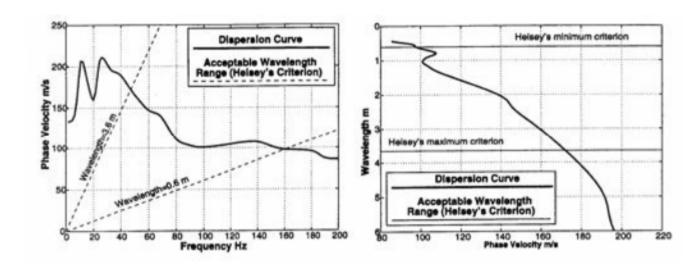






Near Field/Far Field Effect

Recommendation on lamda range based on receiver array length



$$\frac{D}{3} < \lambda < 2D$$













1-page Quick Reference Guide

Receiver spacing: determines the minimum wavlength (λmin) which can be resolved.

$$k_{\text{max}} = \frac{1}{\lambda_{\text{min}}} = \frac{1}{\Delta x}$$

Array total length: theoretically no upper limit on λ_{max} , but because of several effects (ex. presence of noise, higher mode interference, non-ideal digitizers, etc), often choose λmax as:

$$\frac{D}{3} < \lambda_{\text{max}} < 2D$$

Easy to remember: $\lambda_{\text{max}} = \frac{D}{2}$

$$\lambda_{\text{max}} = \frac{D}{2}$$

Near field: need to avoid interference of body waves. Rule of thumb, record at distance λ / 2 for normally dispersive soils, or 2 λ for inversely dispersive soils.













Nonlinear Arrays (brief summary)





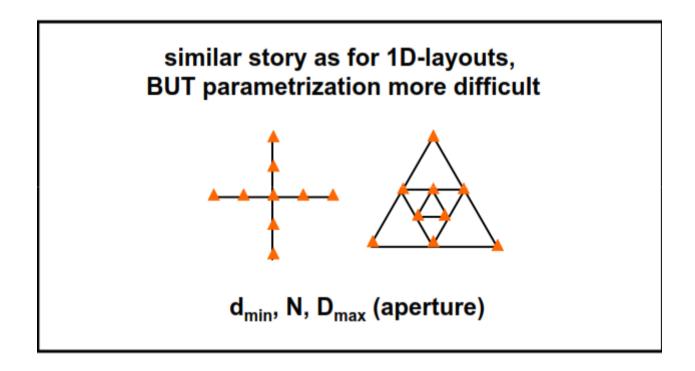








Extension to 2D arrays: finite and discrete arrays











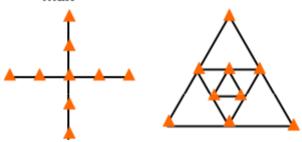




Extension to 2D arrays: finite and discrete arrays

N clear

BUT: d_{min} and D_{max} show directional dependence



especially there will always be some direction, in which d_{min} is vanishing!

limits of array geometry: $\lambda_{min} > 2d_{min}$, $\lambda_{max} \sim 3D_{max}$













Nonlinear Surface Array Methods

- -Cross or circular passive MASW
- -SPAC Spatial Autocorrelation Method
- -High Resolution Frequency Wavenumber Analysis (HRFK)
- -Seismic Interferometry (Controlled Source and Passive)



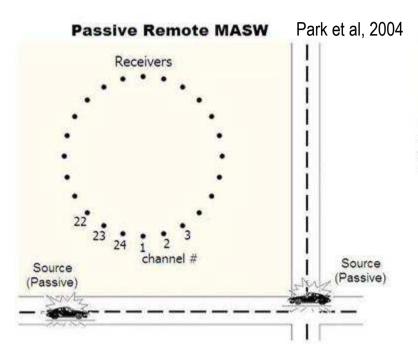


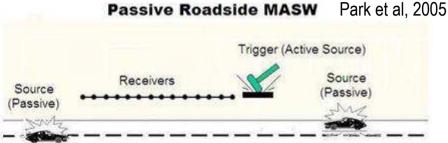












Contrary to ReMi (Louie, 2001), passive remote MASW and passive roadside MASW, require shorter time records with preferably "one" noise source.

"...a detailed study comparing each different type of array and its effect on dispersion analysis has not been reported yet, as far as systematic and scientific perspectives are concerned."











SPAC (Spatial Autocorrelation Method)

Spatial correlation function:

$$\phi(r,\varphi) = \frac{1}{T} \int_{0}^{T} \mathbf{u}(x,y,t) * \mathbf{u}(x+r\cos\varphi,y+r\sin\varphi,t) dt$$

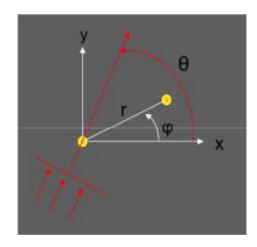
Based on cross-spectrum:

$$\phi(r,\varphi) = \frac{1}{\pi} \int_{0}^{\infty} \Phi(\omega) \cos \left[\frac{\omega r}{c(\omega)} \cos(\theta - \varphi) \right] d\omega$$

Use a narrow band filter:

$$\Phi(\omega) = \Phi(\omega_0)\delta(\omega - \omega_0)$$

$$\phi(r, \varphi, \omega_o) = \frac{1}{\pi} \Phi(\omega_0) \cos \left[\frac{\omega_0 r}{c(\omega_0)} \cos(\theta - \varphi) \right]$$



Correlation coefficient:

$$\rho(r, \varphi, \omega_o) = \frac{\phi(r, \varphi, \omega_0)}{\phi(0, \varphi, \omega_0)} = \cos \left[\frac{\omega_0 r}{c(\omega_0)} \cos(\theta - \varphi) \right]$$



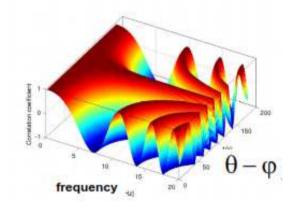








SPAC (Spatial Autocorrelation Method)



$$\rho(r, \varphi, \omega_o) = \cos \left[\frac{\omega_0 r}{c(\omega_0)} \cos(\theta - \varphi) \right]$$

Using two sensors, recording a single plane wave with back-azimuth θ (except θ - ϕ = π /2) we can obtain $c(\omega 0)$

Next compute an azimuthal averaging of spatial correlation coefficients.

$$\bar{\rho}(\mathbf{r}, \omega_0) = \frac{1}{\pi} \int_0^{\pi} \rho(\mathbf{r}, \varphi, \omega_0) \mathbf{d}(\theta - \varphi)$$

$$\overline{\rho}(r,\omega_0) = J_0(\frac{\omega_0 r}{c(\omega_0)})$$



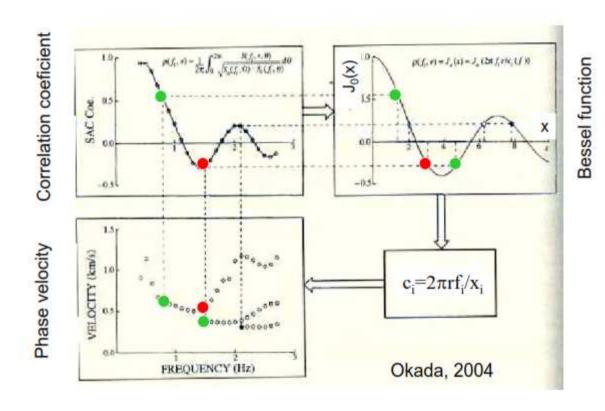








SPAC (Spatial Autocorrelation Method)









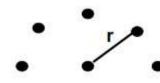




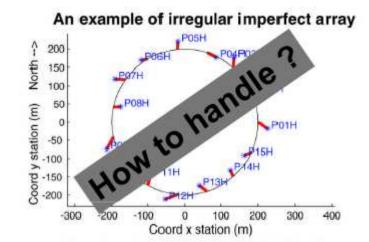


SPAC to MSPAC (Modified SPAC)

SPAC (Aki, 1957)



$$\overline{\rho}(r,\omega_0) = J_0(\frac{\omega_0 r}{c(\omega_0)})$$



Compute azimuthal and radial averaged spatial autocorrelation coefficients:

$$\rho(\omega_o) = \frac{2c_R(\omega_0)}{r_2^2 - r_1^2 \omega_0} \left[r J_1 \frac{\omega_0 r}{c_R(\omega_0)} \right]_{r_1}^{r_2}$$

with r₁ inner radius (controls non-uniqueness), r₂ outer radius (controls resolution)













High Resolution Frequency Wavenumber (HRFK)







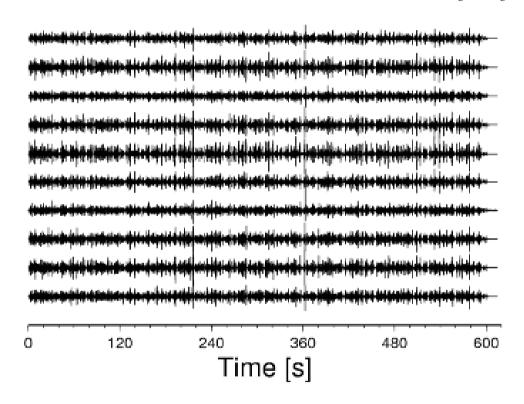






Based on beamforming concept

Question: Is there a signal with parameters θ_o, p_o ? Lets try!







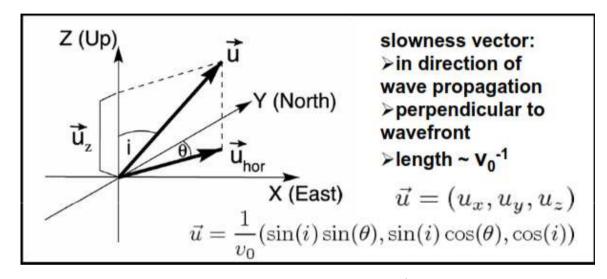








Slowness vector (again)



The slowness vector: $\overrightarrow{u} = u_{hor}(\sin(\theta), \cos(\theta), \frac{1}{\tan(i)})$

Horizontal slowness: $p = |\overrightarrow{u}_{hor}| = \frac{\sin i}{V_o}$

Source: Geopsy H/V tutorial seminar, Thessaloniki, 2010 organized by LGIT, ITSAK, Universitat Postdam, IRD













Delay and sum beamforming

Answer: let's try!

$$\vec{u} = u_{hor}(\sin(\theta), \cos(\theta), \frac{1}{\tan(i)})$$

observation

$$x_i(t) = s(t - ec{r_i}ec{u}_{hor}) + n_i(t)$$

delay observation

$$ilde{x}_i(t) = x_i(t + ec{r}_iec{u}_{hor})$$

$$ilde{x}_i(t) = s(t) + n_i(t + ec{r}_iec{u}_{hor})$$

$$ilde{x}_i(t)=s(t)+n_i(t+ec{r}_iec{u}_{hor})$$
 and sum $b(t)=rac{1}{N}\sum_{i=1}^N ilde{x}_i(t)=s(t)+rac{1}{N}\sum_{i=1}^Nn_i(t+ec{r}_iec{u}_{hor})$

uncorrelated noise is suppressed by \sqrt{N} (at best)





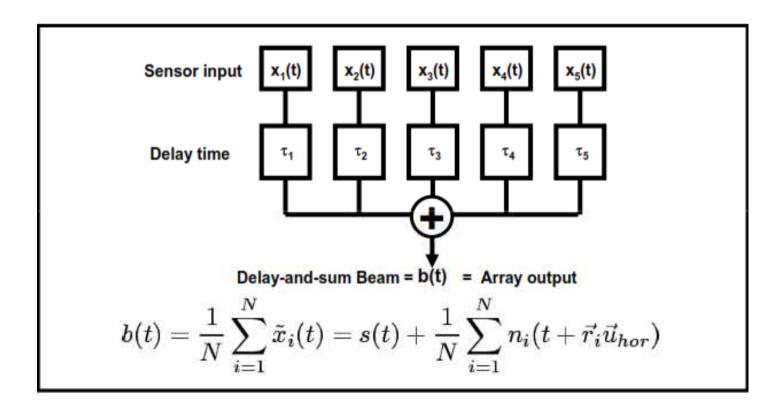








Delay and sum beamforming







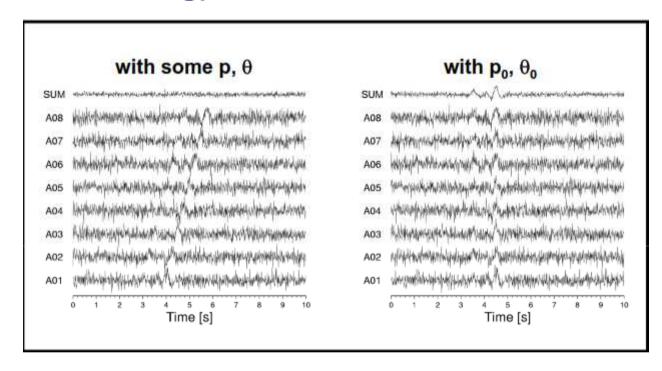








Beam Energy



How well does beamforming work?

Look at beam energy (power measure): $E_{(beam)} = \sum_{k=1}^{N} |b(k\Delta t)|^2$





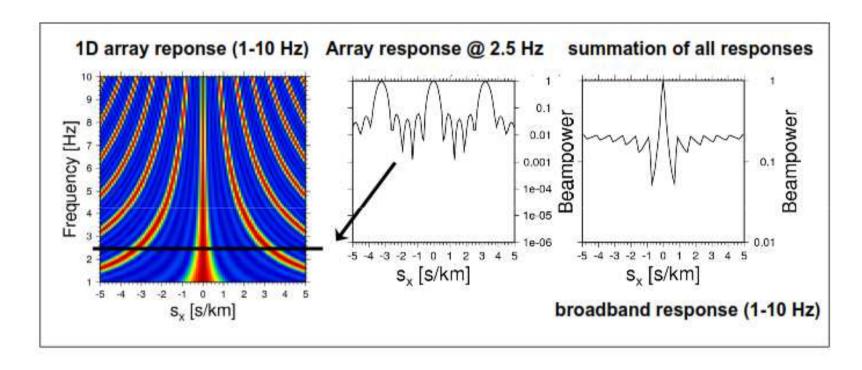








Beam Energy



Example of beam energy spectrum.













'Advanced' Beamforming

<u>Problem</u> with traditional beamforming: what happens if one trace is contaminated by strong noise??

Solution:

Use spectral based beamforming (Barlett, Capon, MUSIC)

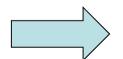
First Fourier transform time to frequency.

Then compute spatial covariance matrix.

Idea is to apply complex weights to sensors equivalent to spatial tapering, and to compute optimum weights

Weights are not computed explicitly, but contained in covariance matrix.

Other example of advanced beamforming are parametric beamformers which include penalty functions used to the spatial covariance matrix.



Product of beamforming is estimate for **f-k spectrum**













Concept:

Assume two recordings at A and B: $r_A(t)$ and $r_B(t)$



Cross-correlation is 'equivalent' to the Green's function of the direct wave between A and B

In other words, like having a source at A and record at B



$$\int_{-\infty}^{+\infty} r_A^*(t) r_B(t+\tau) d\tau$$







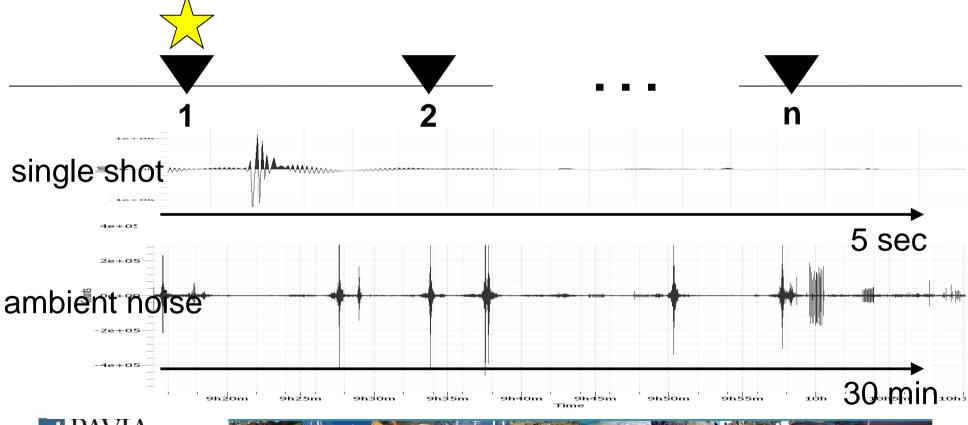






Cross-correlation allows us to take a noise gather, and through cross-correlation, 'imitate' an active shot gather.

Once, we have the 'active' gather, we then compute f-k spectrum









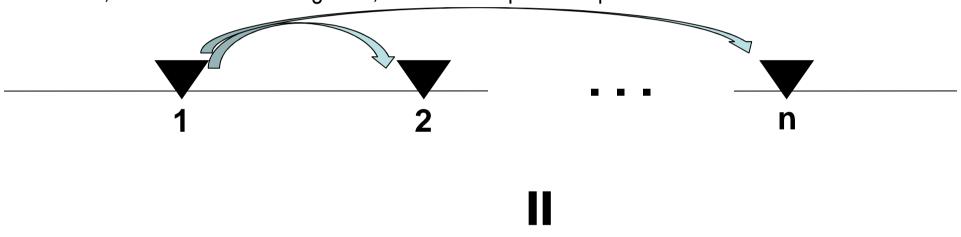






Cross-correlation allows us to take a noise gather, and through cross-correlation, 'imitate' an active shot gather.

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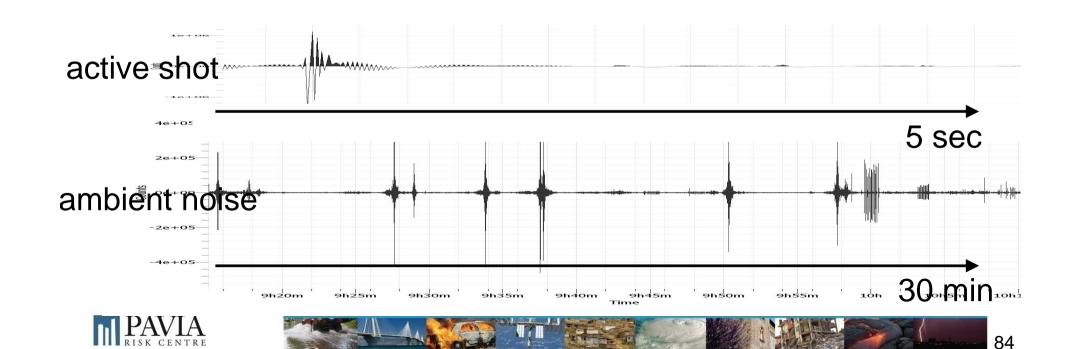






Next page shows a comparison between the active shot gather,

and the "active" shot gather (constructed from cross-correlation of ambient noise)

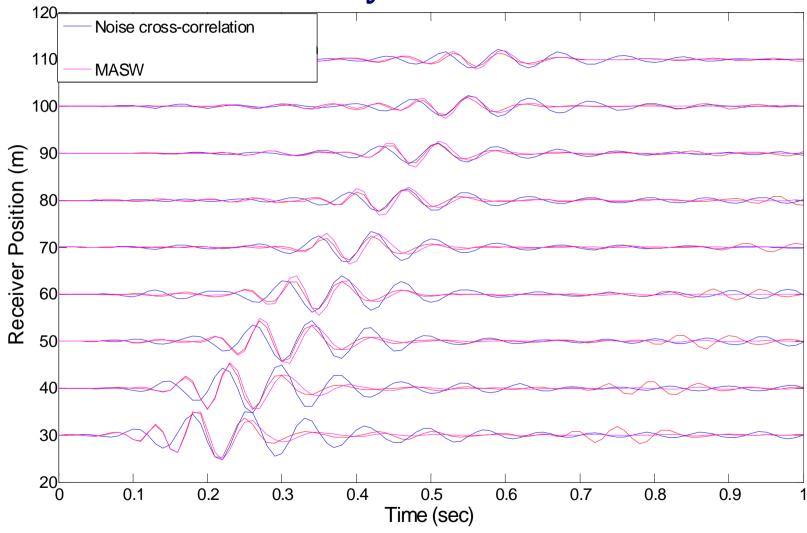
















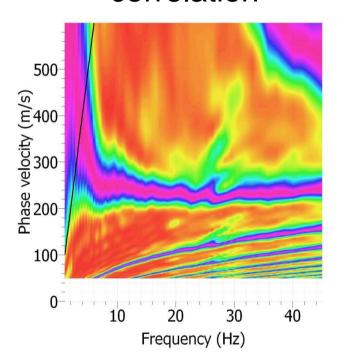




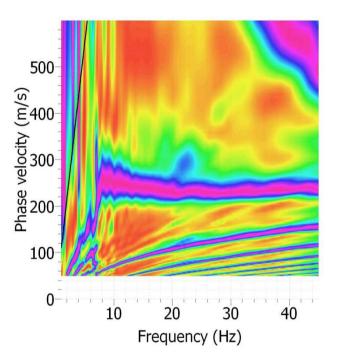




(a) Noise crosscorrelation



(b) MASW



Similar f-k spectra from noise and active source!













Sum up differences between methods

<u>Source: active vs. passive</u> Active is more straighforward, but need good S/N, hence low ambient noise levels. Passive source methods can be advantageous in busy urban settings, but require certain assumptions for noise characteristics (stationarity in time and space). Deviation from these noise characteristics may produce less accurate results.

<u>Array: linear vs. 2D</u> Linear is more straightforward in terms of analysis. However, 2D coverage is often preferred specifically for ambient noise arrays given that it provides insight on noise azimuthal distribution. Also not always practical to have line arrays.

<u>Conclusion:</u> There is no 'best' method. Each method has its limitations, advantages and disadvantages. Question of objectives and logistics.

All methods ultimately seek to construct dispersion curve, and after that... the story (inversion) is the same!!







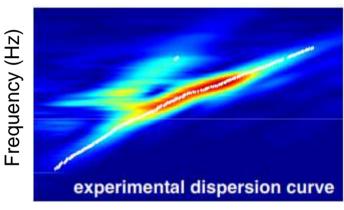






f-k spectrum (1D array) to dispersion curve

f-k spectrum 1D array

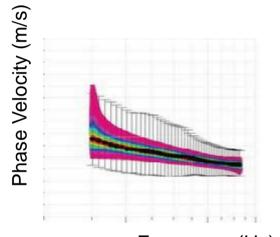


Wavenumber (rad/m)

$k = \frac{2\pi f}{V}$



Dispersion curve



Frequency (Hz)





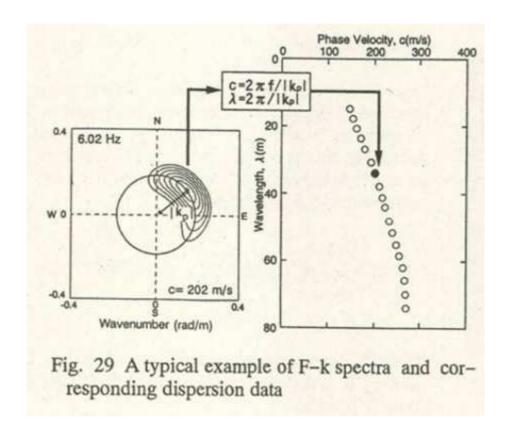








f-k spectrum (2D array) to dispersion curve



Source: Tokimatsu et al., 1997





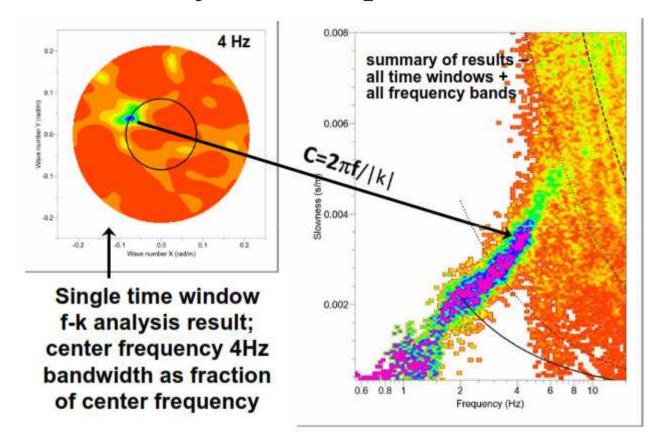








From f-k analysis to dispersion curve















From the dispersion curve to the stiffness profile





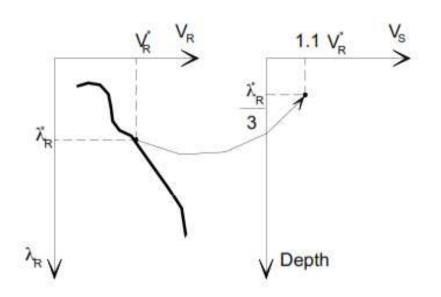


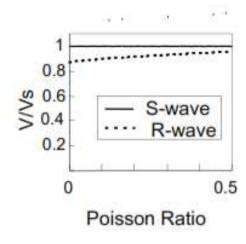






An inversion by 'hand' (quick and approximate)

















A (quick and approximate) $V_{s,30}$ estimate

Based on empirical relationships, can use V_R at a specific lamda of the dispersion curve, and connect it to a $V_{S,30}$ estimate

Model Name	Lamdas involved	formula	
Vs30_MD	40 m	Vs30=1.045*Vr(λ=40m)	
Vs30_AG	40 m	Vs30=Vr(λ=40m)	
Vs30_Co	45 m	Vr(λ=45m)=0.9926*Vs30+23.24	
Vs30_CS	37 m	Vs30=(VR(λ=37m)/0.78)0.97	
Vs30_CB	51 m	Vs30=1.21*(VR(λ=51m))0.96	

Source: Cadet et al, 2011







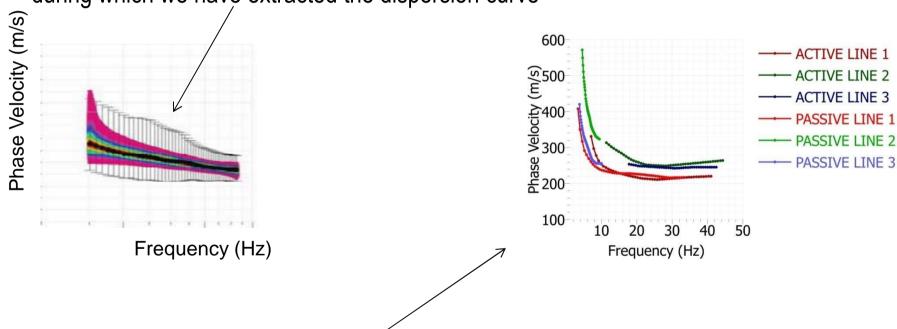






Vs Inversion

Select the dispersion curve along with error approximation (eg s.d.), which directly reflects the quality of the recordings, and our confidence in the processing process during which we have extracted the dispersion curve



Are we combining curves? For example from different lines, from different methods?













Vs Inversion

Select inversion algorithm-- ex least square (damped, weighted, Occam's algorithm) or global search (genetic, neuronal, PSO) etc

Identify a priori information which can be used as constraints (borehole information, other studies?)

Select # of layers, layer thickness, and possible ranges for density, Vp, and Vs # layers should be somehow linked to our knowledge of the geology (not necessarily better to increase # layers).

Layer thickness: (Lmin>dx/3, Lmax<D)

Select an initial model (if no idea, just guess!)

Invert and conquer! Watch scatter in output models, and select/average based on 'judgement'





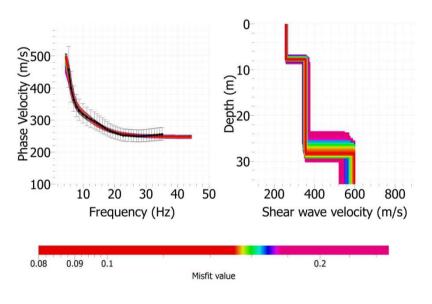








Example of inversion for the same dispersion curve and different number of layers using the Neighborhood algorithm



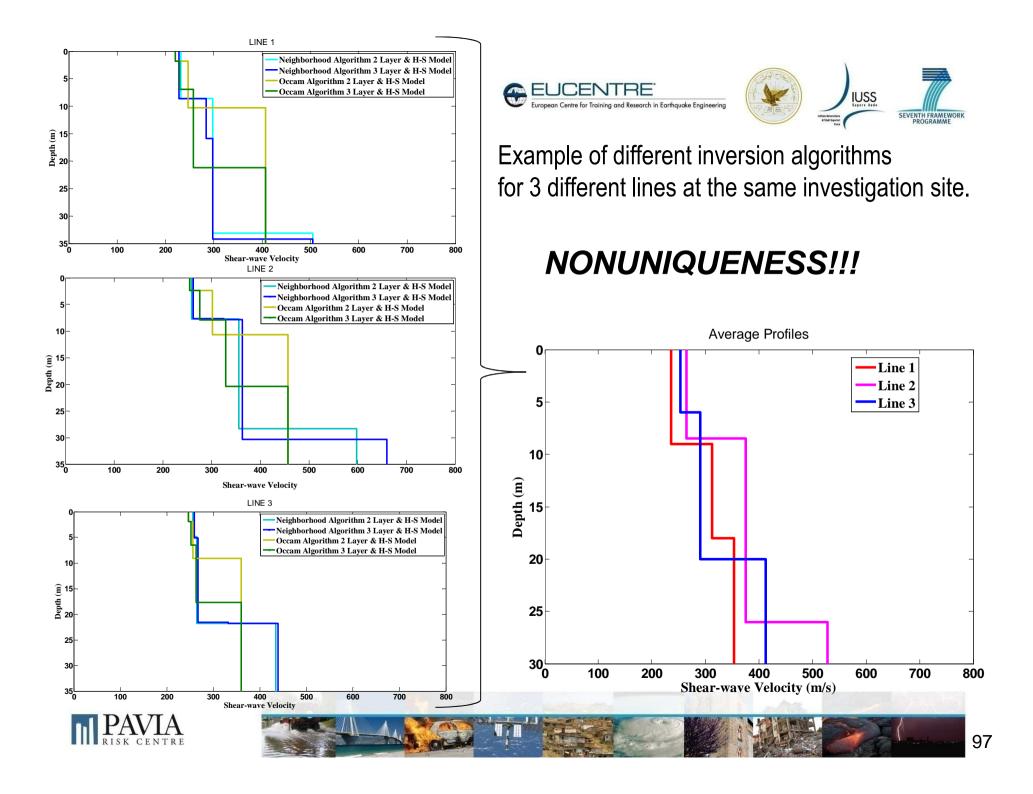
2 layer and half-space

3 layer and half-space

NONUNIQUENESS!!!





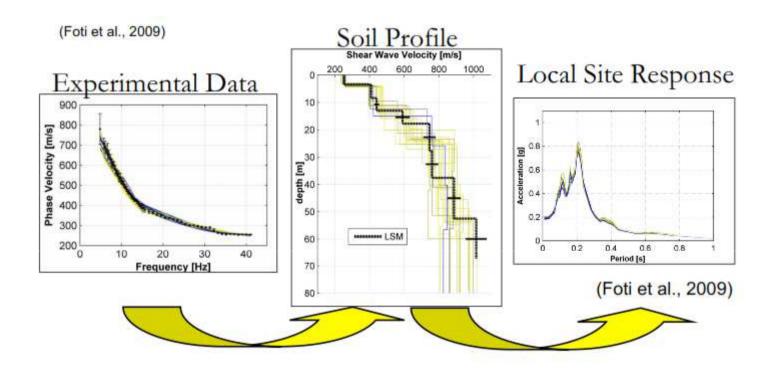












NONUNIQUENESS!!! But, limited consequences in site response analysis as long as similar VS,30 values













From Vs inversion to Vs,30 estimate

$$V_{S,30} = \sum_{i=1,N} \frac{30}{\frac{d_i}{V_{Si}}}$$

EC8 Soil classification

Soil class	V _{s,30}	
Α	> 800	
В	360 - 800	
С	180 - 360	
D	< 180	
E (C, D su A)		

Previous example:

Seismic Line	1	2	3	Average
V _{S,30} (m/s)	297	348	312	319







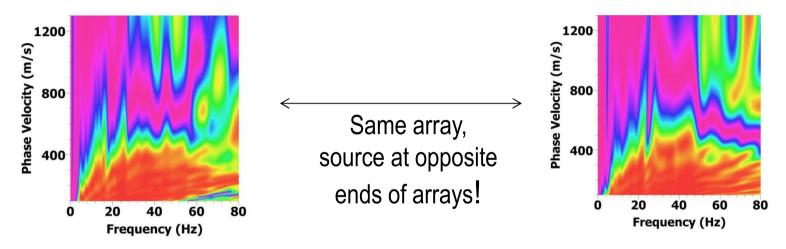






Lateral Heterogeneity

Because inversion is based on **1D** assumption, if site varies considerably laterally at the scale of the investigation (for example the array length), then we cannot invert!



Performing 2D/3D analysis means we at least honor the 1D assumption locally, and observe the smooth changes as we move the investigation target.







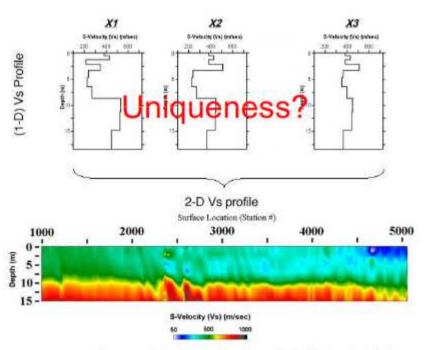




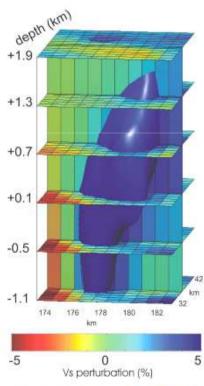


Lateral Heterogeneity

Inversion **based** on 1D assumption. But, we can still go to 2D/3D







D'après Brenguier et al, GRL, 2007





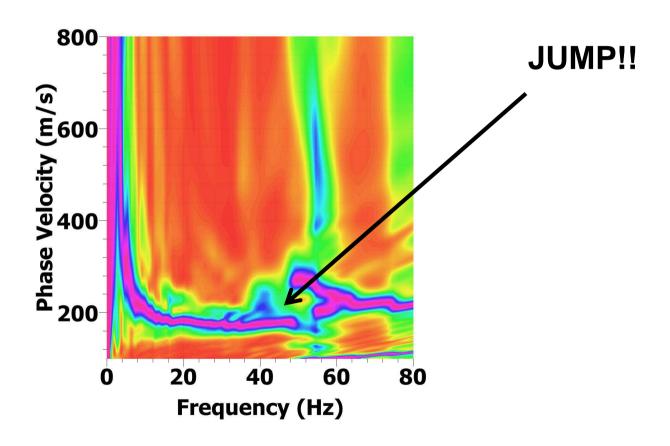








Higher Modes







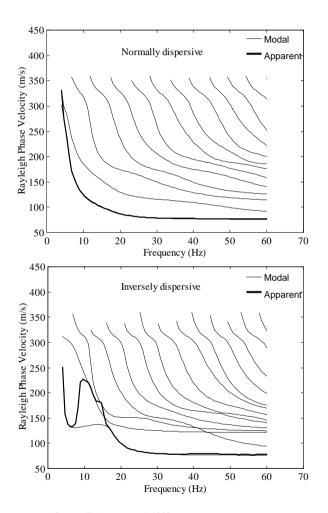
Higher Modes











If the medium is not normally dispersive, higher modes may start to dominate.

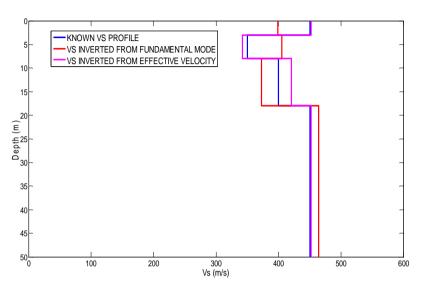
Possible solution: instead of inverting for the fundamental mode, invert for the apparent (effective) dispersion curve.

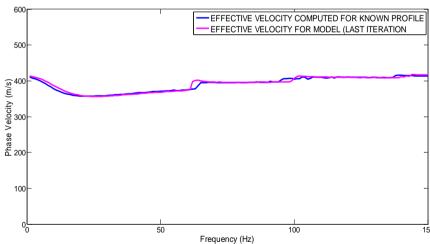
Source: Tokimatsu et al, 1992





Higher Modes













Lai et al, in preparation

Inversion based on effective dispersion curve... maybe this is the right direction!





Inversion of Qs and Vs © EUCENTRE European Centre for Training and Research in Earthquake Engineering









For vertically inhomogeneous media (under weak dissipation assumption):

$$\left[V_{R}\left(\omega\right) = V_{R}^{e} + \left\{\int_{0}^{\infty} V_{S} \left[\frac{\partial V_{R}}{\partial V_{S}}\right]_{\omega, V_{P}} \left[1 - \frac{V_{S}^{e}}{V_{S}}\right] dx_{2} + \int_{0}^{\infty} V_{P} \left[\frac{\partial V_{R}}{\partial V_{P}}\right]_{\omega, V_{S}} \left[1 - \frac{V_{P}^{e}}{V_{P}}\right] dx_{2}\right\}$$

$$\begin{cases}
\alpha_{R}(\omega) = \frac{\omega}{\left[V_{R}(\omega)\right]^{2}} \cdot \left\{ \int_{0}^{\infty} V_{S} D_{S} \left[\frac{\partial V_{R}}{\partial V_{S}}\right]_{\omega, V_{P}} dx_{2} + \int_{0}^{\infty} V_{P} D_{P} \left[\frac{\partial V_{R}}{\partial V_{P}}\right]_{\omega, V_{S}} dx_{2} \right\}
\end{cases}$$





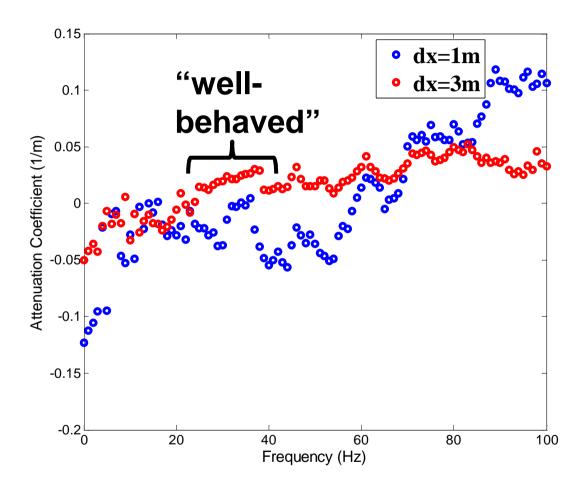
Inversion of Qs and Vs











Example of Rayleigh wave attenuation data





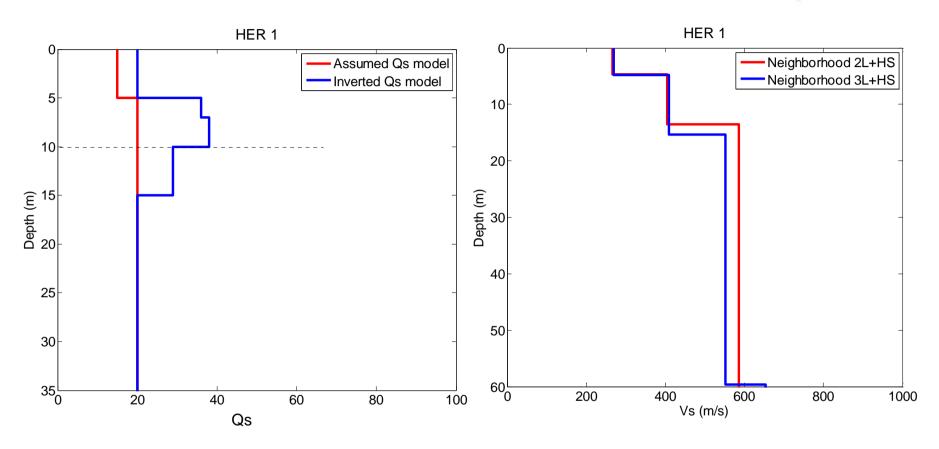
Inversion of Qs and Vs











Same dataset, but Vs inversion penetrates much deeper than Qs inversion. Qs inversion is difficult!













Now some synthetic and real data examples!



