

Basics of signal processing, design of specimens, system acquisition

Scaling strategy

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Background

The range of civil and seismic engineering problems is wide and complex.

Increasingly sophisticated theories and models have emerged to solve problems we face almost daily.

The validity and the predictive power of these models have to be assessed via experimental work.



Principal testing methods

The more common testing methods in the fields of civil and earthquake engineering are

- Quasi-static loading testing method;
- Shaking table testing method;
- Effective force method;
- Pseudo-dynamic testing method;
- Real time pseudo-dynamic testing method;
- Real time dynamic hybrid testing method;
- Centrifuge tests.

Despite the selected testing method, forces or displacements are always applied to the specimen to investigate its behaviour.



Types of Experimental Testing

Classification according to:

- Size of physical model
- Test medium
- Adopted technique

Test types:

- Full-scale field tests
- Small prototype field tests
- **Small/Large scale laboratory tests ('1-g tests')**
- Small scale centrifuge tests ('N-g tests')

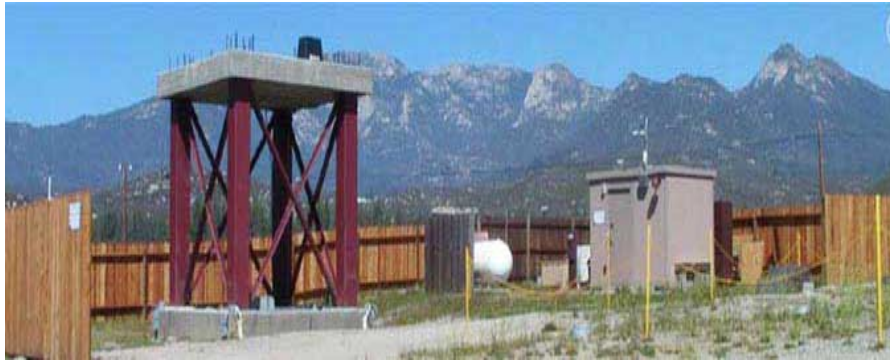
'N-g tests' are very specialised tests in which the gravity is artificially increased using a centrifuge in order to obtain true similarity of stress-strain between the model and the prototype.



Full Scale Field Tests

Permanently Instrumented Field Site for the study of earthquake ground shaking, ground failure, liquefaction and soil-foundation-structure interaction (University of California at Santa Barbara, US)

A number of instruments is used to permanently monitor the site.



Full Scale Field Tests

Modal characterisation (modal periods and damping) of dams, buildings and other structures.

Complete full-scale structures can be tested.

Only low levels of excitation force are possible.

Difficult to explore the non-linear range.

Instruments installed on the structure are used to monitor its behaviour during the excitation.



Full Scale Shaking Table Testing

Specimen: full scale 7 storey wooden building, L=7.5m, W=15.0m, H= 23.5m
Hyogo Earthquake Engineering Research Centre, Japan



'E-Defense', 2000 ton, 3-D Full-Scale Earthquake Testing Facility, Miki, Kobe, Japan



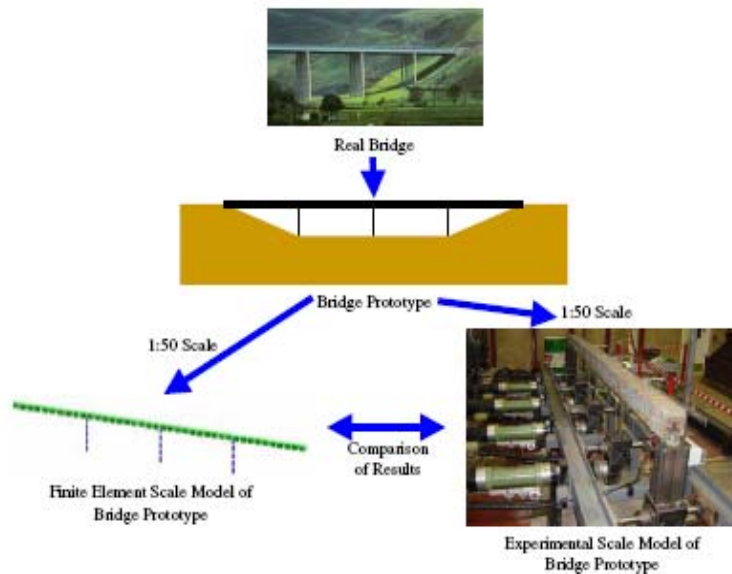
Small Scale Laboratory Tests

('1-g Tests)

- 1:50 Scale Model Bridge installed on a multi-axial support excitation rig
(University of Bristol, UK)



Prototype, Physical Model, FE Model



Principles of physical modelling

Physical modelling reduces the size of a large and complex structure (e.g. building, tunnel, pile foundation) (the 'PROTOTYPE') to a simple structure (the 'MODEL') that maintains the same physical characteristics, the main interaction in the system in order to reproduce the true behaviour of the prototype in a relevant set of loading conditions.

The link between PROTOTYPE and MODEL is governed by analytical expressions between their relevant geometrical and physical parameters ('SIMILITUDE LAWS')

Clearly, a perfect replica model can never be realised in practice because of a number of limitations about design, availability of materials, limitations of our facility in terms of capability and performance. It will be then necessary to accept compromises between the prototype and the model: these compromises should not affect the outcome of the physical modelling



Principles of physical modelling

The approximation of SIMILITUDE LAWS determines the success of physical modelling, model building and testing.

The approximations should respect at least the first order similarities between the prototype and the model, hence they should not cause distortion of the model behaviour.



Benefits and Limitations of Small Scale Modelling

Benefits:

- size reduction, simplification, convenience
- study of systems for which analytical models are too complex (e.g. shells, lattices)
- provides experimental data to validate theoretical models

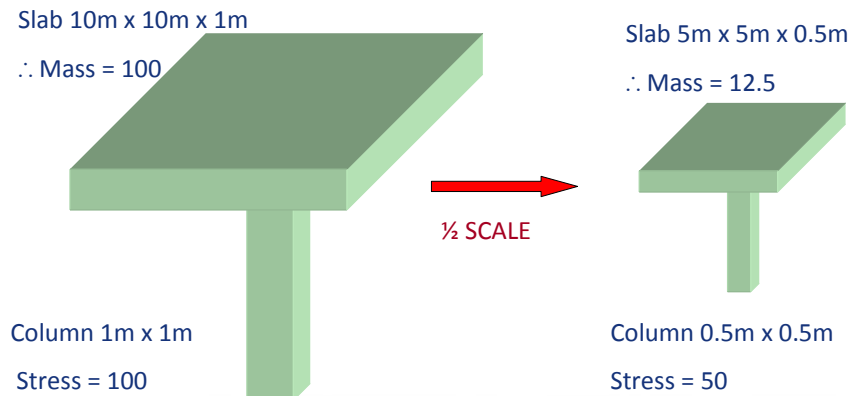
Limitations:

- full compliance with similitude laws is often impossible (the MODEL will be APPROXIMATE)
- capabilities and performance of test facilities influence modelling and design
- risk of unknown relevant interactions not captured in the model
- manufacturing, measurement and testing costs affect modelling and design



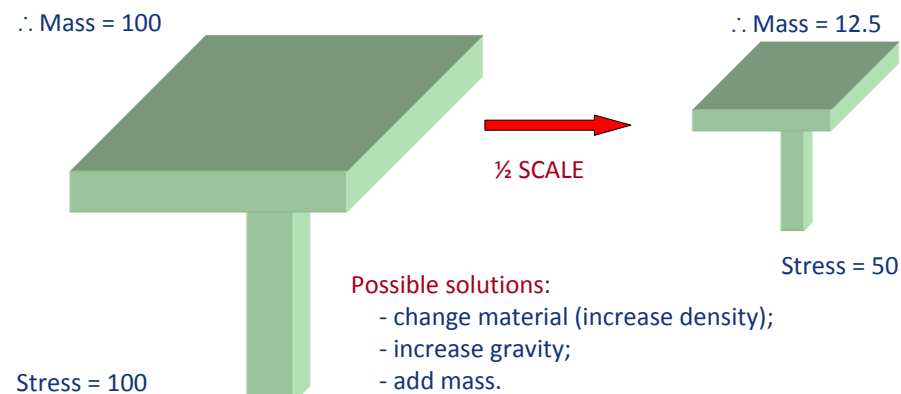
Similitude Laws : The Problem

As an example, let us consider a simple prototype and the 1:2 scaled model.
As 1:2 scale is applied to all the dimensions, also the resultant stress will be half the expected: this will cause problem as our prototype is characterised by a particular stress-strain behaviour of the material.



Similitude Laws : The Problem

Since the model should reproduce relevant stress-strain behaviour of the prototype in any chosen circumstances of loading, there is a need for changing the model in order to restore the proper similarity. There are different possible solutions, leading to different types of tests.



Buckingham's PI Theorem

Any dimensionally homogeneous equation involving certain physical quantities can be reduced to an equivalent equation involving a complete set of dimensionless products (Buckingham 1914).

$F(X_1, X_2, \dots, X_n) = 0$ where 'n' is the number of relevant parameters in the system
(e.g. 'length', 'time', 'gravity', 'mass density', 'stress', etc)

Can be reduced to:

$G(\pi_1, \pi_2, \dots, \pi_m) = 0$ where 'm' is the number of dimensionless products

$m = n - r$, where 'r' is the number of fundamental measures (e.g. 'force', 'length', 'time')

(See ref: 'Structural Modeling and Experimental Techniques' by G. Sabnis, H. Harris, R. White, M. Saeed Mirza, Prentice Hall Intl, NJ, 1983)



Scaling of geometric quantities

When dealing with Geometry, the only physical quantity to deal with is the length, as the angles are dimensionless.

The creation of a scaled figure is intuitive: it will be enough to create a model figure ω similar to the prototype Ω . Then if L is a given length in Ω , the corresponding scaled one will be l in ω . The quantity λ , ratio between L and l is the similitude ratio, constant for all the length.

In the same way, if A is an area in Ω , the same area in the model ω will equal a :

$$a = \lambda^{-2} A$$

Since the same proportion applies to all the linear dimensions.

When dealing with volumes, clearly a volume V in the prototype Ω will equal the product of the volume v (in the model ω) and the cube of the scaling ratio λ .

$$V = \lambda^3 v$$



Scaling of geometric quantities

On the other hand, since angles are dimensionless, the value of the angles measured in the scaled model ω will equal the value of the same angles in the prototype Ω .

Generally speaking, if a quantity D in the prototype Ω has a dimension α with respect to the length, it can be derived by the scaled quantity d of the model ω using the following relation:

$$D = \lambda^\alpha d$$



Scaling of kinematic quantities

Fundamental quantities in kinematic problems are length and time.

Also in this case the construction of a scaled model ω of a kinematic mechanism Ω is quite intuitive. The model should be geometrically similar to the prototype for all the possible configuration and its movements should last a time t function on the duration T of the movements of the prototype, t and T should respect the relation:

$$T = \tau t$$

where τ is constant for all the movement of the kinematic mechanism.

Within this kinematic similarity the velocity U of a point of the prototype Ω can be evaluated based on the velocity u of the model ω as:

$$U = \lambda \tau^{-1} u$$

Analougly, for the acceleration:

$$G = \lambda \tau^{-2} g$$



Scaling of kinematic quantities

In general, if a kinematic quantity D of the prototype mechanism Ω has dimension α with respect to the length and dimension β with respect to the time, it can be derived starting from the quantity d of the model mechanism ω using the relation:

$$D = \lambda^\alpha \tau^\beta d$$

Worth to mention is that this kind of models does not have a large application for our purposes as geometric and mechanical models are the more widely used.



Scaling of mechanical quantities

Fundamental quantities in mechanical problems are length, mass and time; all the other quantities are normally derived by combining these three fundamental ones.

To realise a model ω of a mechanical prototype Ω , the model should satisfy all the requirements relative to the kinematic model and additionally if M is a mass of the prototype Ω and the corresponding mass m of the model ω should respect the relation:

$$M = \mu m$$

where μ is constant for all the masses.

In general, if a mechanical quantity D of the prototype mechanism Ω has dimension α with respect to the length, dimension β with respect to the time and dimension γ with respect to mass, it can be derived starting from the quantity d of the model mechanism ω using the relation:

$$D = \lambda^\alpha \tau^\beta \mu^\gamma d$$



Scaling of mechanical quantities

In general, if a mechanical quantity D of the prototype mechanism Ω has dimension α with respect to the length, dimension β with respect to the time and dimension γ with respect to mass, it can be derived starting from the quantity d of the model mechanism ω using the relation:

$$D = \lambda^\alpha \tau^\beta \mu^\gamma d$$

As an example, measuring a force f on the model ω , knowing that for a force:

- $\alpha = 1$;
- $\beta = -2$;
- $\gamma = 1$;

the force F on the prototype is:

$$F = \lambda \tau^{-2} \mu f$$

This relation states the force similitude.

Nevertheless, it has to be considered that normally the scaling ratio λ and μ (respectively for length and mass) are related: $\mu = \lambda^3$



Newton rule

Let us consider a model ω is realised using the same materials (Hp#1) of the prototype Ω , and the gravity loads as a spontaneous force (Hp#2).

If these two hypothesis are true, as in the majority of the testing activities, the gravity loads will be due to the model masses, which in turn are proportional to the volumes, hence the mass ratio and the length ratio will respect the relation:

$$\mu = \lambda^3$$

The second hypothesis implies that the weights (spontaneous forces) must respect the relation:

$$F = \lambda^3 f$$

where F and f are the forces acting respectively on the prototype Ω and on the model ω



Newton's rule

Then, it has to be considered that in general:

$$F = M G = (\mu m) (\lambda \tau^2 g) = (\mu \lambda \tau^2) mg = \mu \lambda \tau^2 f$$

Joining the three previous equations, it derives that $\mu \lambda \tau^2 = \lambda^3$ or:

$$\lambda \tau^2 = 1$$

This condition states that the accelerations on the prototype equals the accelerations on the model: as reasonable, in pseudo-static, pseudo-dynamic and shake table tests the gravity acceleration acting on model and prototype is clearly the same.

Furthermore, from the previous relation derives that the time scale and the length ratio are related:

$$\tau = \lambda^{1/2}$$

This implies that during a shake table test, the duration of the seismic motion will be scaled (the scaled accelerogram will be faster than the original).



Newton's rule

Going back to the previous relation between a generic quantity d of a model ω and the corresponding one of the prototype Ω , including the previous equation it is possible to obtain:

$$D = \lambda^\alpha \tau^\beta \mu^\gamma d = \lambda^\alpha \lambda^{\beta/2} \lambda^{3\gamma} d = \lambda^{\alpha + \beta/2 + 3\gamma} d$$

This is the Newton's rule. Worth to mention is that for this to be valid all the spontaneous forces (such as the weight) must be scaled as the volumes.

Additionally it has to be noted that, applying the Newton's rule, all the stresses σ acting on the model ω will be scaled with a factor equal to λ with respect to the stresses Σ acting on the unscaled prototype Ω .

$$\Sigma = F / A = (\lambda^3 f) / (\lambda^2 a) = \lambda \sigma$$



Newton's rule

One direct implication of having scaled stresses acting on the model is that this scaling method is applicable when the **elastic response** of the prototype is under investigation (e.g. dynamic characterisation).

When investigating the response of the prototype in the **non-linear range** (e.g. shake table tests), scaled stresses in the model will lead to an overestimation of the prototype resistance.

Then, to obtain useful / truthful results, some modifications of the scaled model are required: in particular, for most of laboratory testing (i.e. with no modification to the gravity acceleration), additional masses are required to restore the proper stress level within the model.

It is crucial that these additional masses modify only the model mass having NO-influence on neither dimensions nor stiffness.



Scaled model design

As already mentioned, when testing the behaviour of a structure using a scaled model, modifications to the model should be adopted to obtain a proper behaviour reflecting the real structural response.

In seismic testing, there are a number of issues to be considered in the model design phase:

- all the materials of the scaled model ω are normally the same as those of the prototype Ω ;
- the scaling ratio between the stress of the model (σ) and those of the prototype (Σ) must equal 1;
- the weight must be a spontaneous force (1-g tests).



Scaled model design

As an example, let us consider a prototype building 20 m high and a model with height equal to 5 m.

In this case the length ratio λ equals 5.

To have the ratio between the stresses equal to 1, the ratio between the forces should be equal to λ^2

$$\Sigma = F / A = \text{forceRatio } f / \lambda^2 a = \text{forceRatio} / \lambda^2 \sigma$$

So, to have $\Sigma = \sigma$, the force ratio must equal λ^2

But the weight (P) acting on the prototype and the weight (p) of the model are in the ratio of λ^3 (like the volumes).

From here the necessity to add a fictitious weight (p^*) to the model in order to restore the λ^2 ratio between the gravity loads:

$$P = \lambda^3 p = \lambda^2 (p + p^*)$$



Scaled model design

The previous equation leads to the determination of the additional weight:

$$\lambda^2 = (p + p^*) / P = (p + p^*) / (\lambda^3 p)$$

and solving for p^* :

$$p^* = (\lambda - 1) p$$

This relation expresses the relation between the additional weight and the weight of the model.

It has to be noted that these additional masses are NOT negligible: for the example case in which $\lambda = 5$, the additional masses to be installed on each floor equal four times the weight of the floor itself (neglecting the weight of the vertical elements).



Scaling factor

The scaling factor λ to be adopted for a dynamic shake table test must be chosen considering the following issues:

- platen dimension: the scaled model must fit on the shake table to assure proper fixing to it;
- maximum payload and overturning moment: the weight of the scaled model must not exceed the maximum payload and overturning moment (also considering the desired acceleration);
- height of the lab: the scaled model must not interfere with the roof of the lab;
- dynamic performance of the actuation system: low scaling factor involves high masses, high scaling factors involves scaling of time and consequently high frequency content of the imposed motion;
- need for application of additional masses: the higher the scaling factor, the high the additional masses (consider different heavier materials).

It has to be noted that the additional masses must be connected to the model structure in order to experience the same acceleration of the model. On the other hand, such fixing system must avoid improper contributions of the additional masses to the structural stiffness and resistance.



Scaling factor

The scaling factor clearly affects all the involved quantities.

Type	Quantity	Symbol	Dimension	Model	Newton's rule	Shake table test
Force	Point	P	[mlt ⁻²]	$\mu \lambda \tau^{-2}$	λ^3	λ^2
	Linear	p	[mt ⁻²]	$\mu \tau^{-2}$	λ^2	λ
	Distributed	q	[ml ⁻¹ t ⁻²]	$\mu \lambda^{-1} \tau^{-2}$	λ	1
	Pressure	q	[ml ⁻¹ t ⁻²]	$\mu \lambda^{-1} \tau^{-2}$	λ	1
	Bending moment	M	[ml ² t ⁻²]	$\mu \lambda^2 \tau^{-2}$	λ^4	λ^3
	Weight	W	[mlt ⁻²]	$\mu \lambda \tau^{-2}$	λ^3	λ^2
	Specific weight	w	[ml ⁻² t ⁻²]	$\mu \lambda^{-2} \tau^{-2}$	1	λ^{-1}
Geometry	Length	L	[l]	λ	λ	λ
	Area	A	[l ²]	λ^2	λ^2	λ^2
	Volume	V	[l ³]	λ^3	λ^3	λ^3
	Rotation	r	[-]	1	1	1



Scaling factor

The scaling factor clearly affects all the involved quantities.

Type	Quantity	Symbol	Dimension	Model	Newton's rule	Shake table test
Materials	Stress	σ	$[ml^{-1}t^{-2}]$	$\mu \lambda^{-1} \tau^{-2}$	λ	1
	Deformation	ϵ	$[-]$	1	1	1
	Elastic Modulus	E	$[ml^{-1}t^{-2}]$	$\mu \lambda^{-1} \tau^{-2}$	λ	1
	Poisson's coefficient	ν	$[-]$	1	1	1
	Mass	M	$[m]$	μ	λ^3	λ^2
	Density	ρ	$[ml^{-3}]$	$\mu \lambda^{-3}$	1	λ^{-1}
Dynamic	Time	t	$[t]$	τ	$\lambda^{1/2}$	$\lambda^{1/2}$
	Frequency	f	$[t^{-1}]$	τ^{-1}	$\lambda^{-1/2}$	$\lambda^{-1/2}$
	Periods	T	$[t]$	τ	$\lambda^{1/2}$	$\lambda^{1/2}$
	Displacement	d	$[l]$	λ	λ	λ
	Velocity	v	$[lt^{-1}]$	$\lambda \tau^{-1}$	$\lambda^{1/2}$	$\lambda^{1/2}$
	Acceleration	a	$[lt^{-2}]$	$\lambda \tau^{-2}$	1	1



Structural Models

- **True Replica Models:** simultaneous duplication of inertial, gravitational and restoring forces (in practice, these models are impossible to achieve).
- **Adequate Models:** maintain 'first order' similarity (i.e. they capture the relevant interactions). Require previous insight into how a system behaves. Only the relevant interactions are modelled.
- **Distorted Models:** do not satisfy one or more of the 'first order' stipulations of dimensional analysis.



Summary of Scaling Factors for Earthquake Response of Structures

Parameter to be scaled (1)	Dims. (2)	Scale factors for particular types of model					
		True replica models		Shaking table models			
		Normal gravity (3)	Artificial gravity (centrifuge models) (4)	Artificial mass simulation (material modelled) (5)		Gravity forces neglected (material modelled) (7)	
				(prototype material) (6)		(prototype material) (8)	
Length (L) *	L	S_L	S_L	S_L	S_L	S_L	S_L
Time (t) *	T	$S_L^{1/2}$	S_L	$S_L^{1/2}$	$S_L^{1/2}$	$S_L S_E^{-1/2} S_\rho^{1/2}$	S_L
Frequency (f)	T ⁻¹	$S_L^{-1/2}$	S_L^{-1}	$S_L^{-1/2}$	$S_L^{-1/2}$	$S_L^{-1} S_E^{1/2} S_\rho^{-1/2}$	S_L^{-1}
Velocity (v)	LT ⁻¹	$S_L^{1/2}$	$S_L^{1/2}$	$S_L^{1/2}$	$S_L^{1/2}$	$S_E^{1/2} S_\rho^{-1/2}$	1
Gravitational Accel. (g) *	LT ⁻²	1	S_L^{-1}	1	1	neglected	neglected
Acceleration (a) *	LT ⁻²	1	S_L^{-1}	1	1	$S_L^{-1} S_E S_\rho^{-1}$	S_L^{-1}
Mass Density (ρ) *	ML ⁻³	S_E / S_L	1	S_E / S_L **	1 / S_L **	S_E	1
Strain (ε)	-	1	1	1	1	1	1
Stress (σ) *	ML ⁻¹ T ⁻²	S_E	1	S_E	1	S_E	1
Young's Modulus (E) *	ML ⁻¹ T ⁻²	S_E	1	S_E	1	S_E	1
Specific Stiffness (E/ρ)	L ³ T ⁻²	S_L	1	S_L	S_L	$S_E S_\rho^{-1}$	1
Deflection (δ) *	L	S_L	S_L	S_L	S_L	S_L	S_L
Force (Q)	MLT ⁻²	$S_E S_L^2$	S_L^2	$S_E S_L^2$	S_L^2	$S_E S_L^2$	S_L^2
Energy (EN)	ML ² T ⁻²	$S_E S_L^3$	S_L^3	$S_E S_L^3$	S_L^3	$S_E S_L^3$	S_L^3
Pressure (q)	ML ⁻¹ T ⁻²	S_E	1	S_E	1	S_E	1
Mass (M)	M	$S_E S_L^3$	S_L^3	$S_E S_L^3$ ***	S_L^3 ***	$S_E S_L^3$	S_L^3
Poisson's Ratio (ν)	-	1	1	1	1	1	1

Note: S = ratio of prototype quantity to model quantity

** Use these parameters when considering distributed mass systems

*** Use these parameters when considering lumped mass system



Scaling for Shaking Table Tests

For scale modelling on a shaking table, there are two categories of modelling:

- artificial mass simulation;
- gravity loads are neglected

Let us see an example, on a shake table we can not modify the gravity force then

$$S_g = 1, \text{ and as } \frac{S_g S_L S_\rho}{S_E} = 1 \quad \begin{array}{l} g: \text{gravity}; L: \text{length}; \\ \rho: \text{density}; E: \text{stiffness} \end{array}$$

$$\text{then: } S_\rho = \frac{S_E}{S_L}$$

PROBLEM: We cannot scale density and stiffness in the same time !!! Then there will be a need for additional masses



Increase Gravity – Centrifuge Testing

If we can change gravity then we can select S_L , S_ρ and S_E as the basic parameters to scale

From
$$\frac{S_g S_L S_\rho}{S_E} = 1$$

Using prototype materials having $S_\rho = 1$ and $S_E = 1$ (hence prototype and model will have the same material)

We get the relation between the scaling factor for gravity and length:

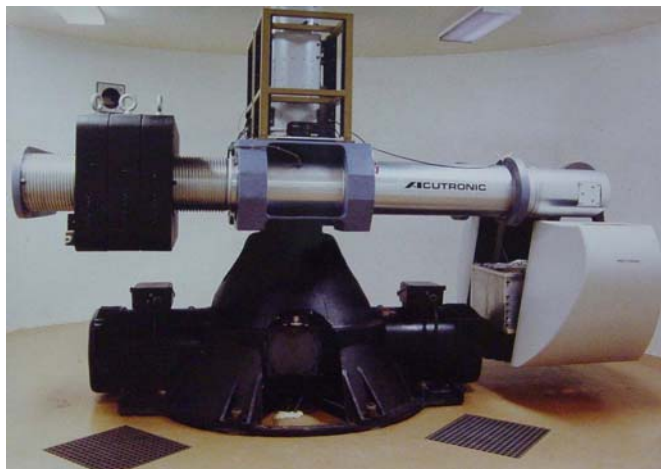
$$S_g = \frac{1}{S_L}$$



Small Scale Centrifuge Tests (‘N-g Tests’)

Geotechnical centrifuge, nominal radius 2.7 m, 160 g at 1.5 ton
(Rensselaer Polytechnic Institute, US)

Centrifuge tests are normally used for investigation regarding soil structure interaction and geotechnical aspects.



Add Mass – Shaking Table Models

If we cannot change gravity then we can select S_L , S_g and S_E as the basic parameters to scale

From
$$\frac{S_g S_L S_\rho}{S_E} = 1$$

And using prototype materials $S_E = 1$ and $S_g = 1$

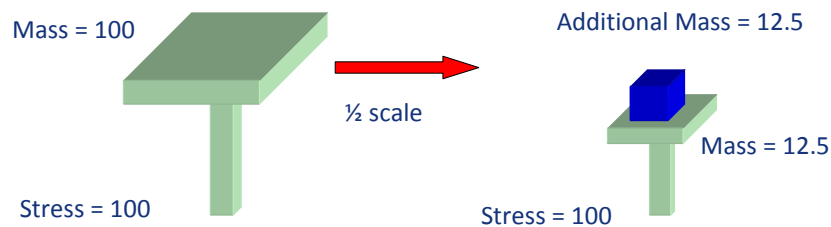
We get
$$S_\rho = \frac{1}{S_L}$$

But don't increase density – instead we add (lumped) mass to increase effective density



Add Mass – Lumped Mass System

Add mass at locations of lumped masses
Additional mass must NOT contribute to stiffness

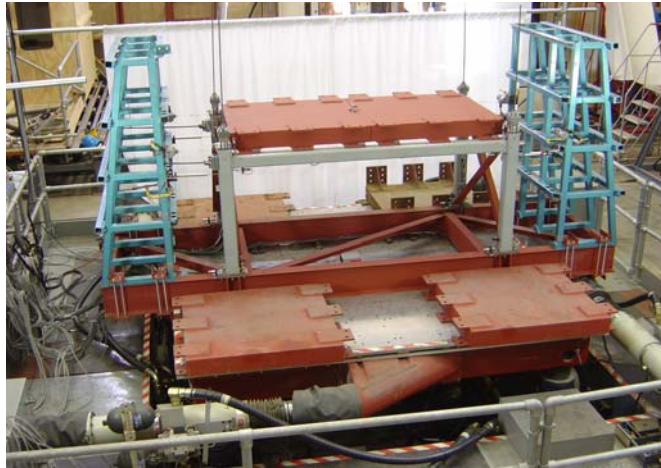


Mass scaling $S_M = S_L^2$



Test with Lumped Artificial Mass

Specimen: scaled nonlinear elastic moment resisting frame, $W=2.1$ m, $H=0.9$ m,
2t mass on top of the frame represents the scaled applied load on the beams
(University of Bristol, UK)



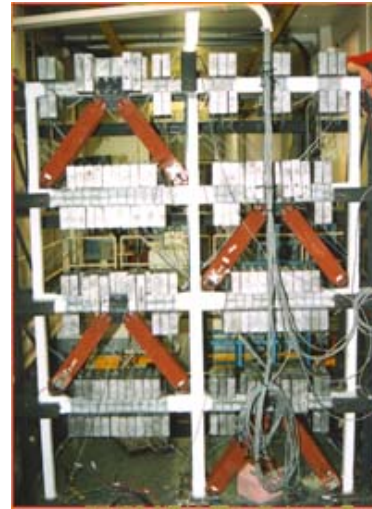
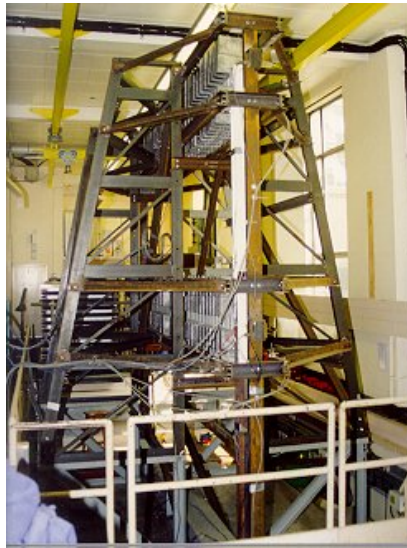
Add Mass – Distributed Mass System

Another path may be possible, this is slightly more difficult than adopting a lumped mass system: a distributed additional mass system can be installed on model of the prototype.

In any case, it is very important that additional masses must NOT contribute to the stiffness of the tested model (this to guaranty that proper behaviour is preserved).



Tests with Distributed Artificial Mass

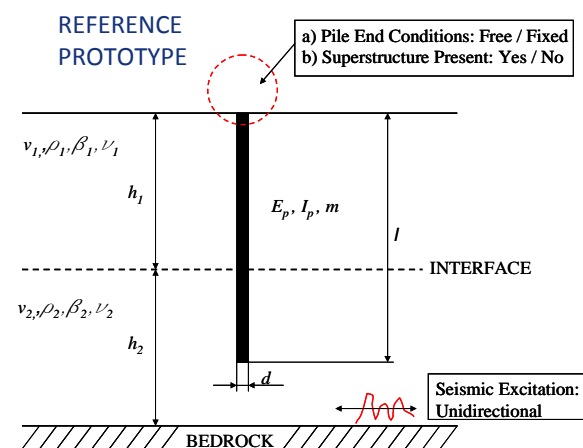


(University of Bristol, UK)



Case Study:

A Physical Model of Soil-Pile-Superstructure Interaction



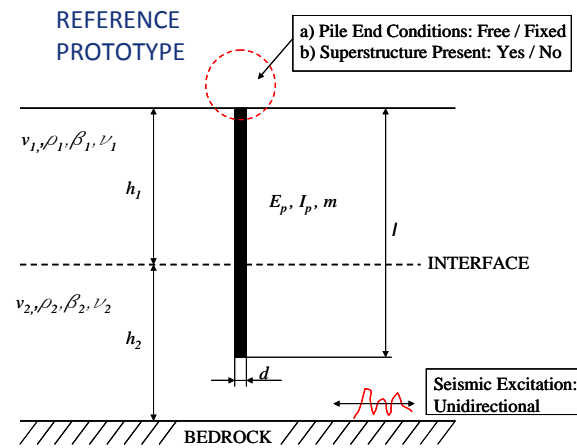
Within the framework of ReLUIS (Italian Network of the University Laboratories of Earthquake Engineering) a model was tested: a concrete pile penetrating a double layer soil with a certain stiffness contrast between the two layers. Different boundary conditions were considered:

- the pile end was free or fixed;
- the superstructure was present or not



Case Study:

A Physical Model of Soil-Pile-Superstructure Interaction



Sequence of Activities in Physical Modelling

For the selected case study, but also in general, the first thing to do is to define objectives:

- Desired outcomes
- Expected interactions
- Expected failure mechanisms

Assess various candidate solutions of models.

Select the best candidate solution for modelling.

Establish similitude laws between Model and Prototype to be able to design the physical model:

- Testing of reduced models (certain important components only)
- Computer simulations & parametric studies



Assessment of Candidate Solutions for Physical Modelling

When different candidate solutions must be checked to choose the one to be realised and tested, different **assessment criteria** must be considered:

- Desired outcomes;
- Degree of representation;
- Feasibility of model (in relation to existing test facilities);
- Manufacturing costs;
- Manufacturing and assembling time;
- Testing time;
- Measurement system (costs, convenience);
- Number of required personnel for handling and testing.



Conceptual Assessment Chart for a Soil-Structure Interaction Model

This chart can be used to evaluate different possible solutions: a score is given to each solution with respect to feasibility and outcomes.

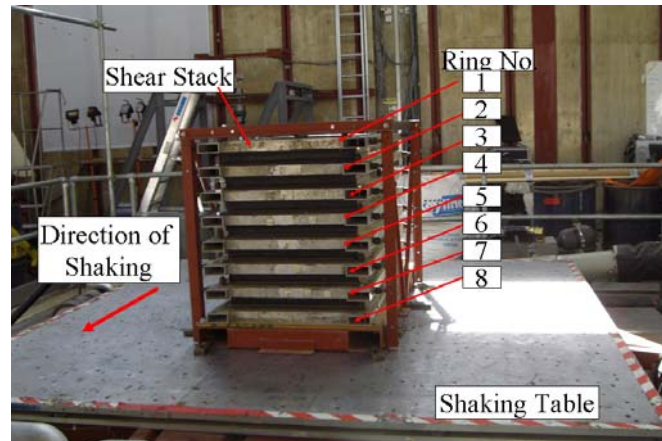
Marks: 0 - poor
1 – satisfactory
2 – good
3 – very good

Model	Feasibility point of view						Outcomes point of view			
	Degree of Representation	Compliance With Similitude Laws	Costs	Manufacturing Time	Measurement Costs	Score	Soil-pile Interaction	Inertial Effects	Soil stiffness degradation	Score
A	3	3	0	0	0	6	3	3	3	9
B	2	2	1	0	1	6	3	2	1	6
C	2	1	2	2	2	9	2	2	0	4
D	1	1	3	3	2	9	2	1	0	3



Test facilities

Laminar large scale shear box
L=1190 mm, W=550 mm, H= 818 mm
 $f_n = 6$ Hz (empty box)



Test facilities



Earthquake Simulator at University of Bristol

- Size: 3 m x 3 m. Axes: 6
- Platform mass: 3.8 tonnes
- **Max. payload: 15 tonnes**
- Max. payload height: 15 m
- Z actuators: 4 at 50 kN
- Vertical acceleration: 5.6 g (no payload)
- Vertical acceleration: 1.2 g (10 tonne payload)
- X and Y actuators: 4 at 50 kN
- Horizontal acceleration: 3.7 g (no payload)
- Horizontal acceleration: 1.6 g (no payload)
- Displacement 3 axes: +/- 150 mm
- Velocity 3 axes: 1.2 m/s
- **Frequency range: 0-60 Hz**

Scaling Laws for 1-g Geotechnical Models

Variable	Scale factor	Magnitude
Length	n_l	n^{-1}
Density	n_ρ	1
Stiffness	n_G	$n^{-0.5}$
Acceleration	n_g	1
Stress	$n_\rho n_g n_l$	n^{-1}
Strain	$n_\rho n_g n_l / n_G$	$n^{-0.5}$
Displacement	$n_\rho n_g n_l^2 / n_G$	$n^{-1.5}$
Velocity	$n_g n_l (n_\rho / n_G)^{0.5}$	$n^{-0.75}$
Dynamic Time	$n_l (n_\rho / n_G)^{0.5}$	$n^{-0.75}$
Frequency	$(n_l)^{-1} (n_\rho / n_G)^{-0.5}$	$n^{0.75}$
Shear wave velocity	$(n_\rho / n_G)^{-0.5}$	$n^{-0.25}$

(See ref: D. Muir Wood, 'Geotechnical Modelling', Spon Press, Oxfordshire, 2004)



Scaling Facts

- The ratio between the prototype soil depth (30 m) and the shear stack height (0.8 m) gave the fundamental scale factor for length ($S_L = 37.5$)
- Stiffness ratio between the soil layers has to be preserved in model and prototype: $G_2/G_1 = 16$. Stiffness design: selection of model soil material and pluviation parameters.
- Shear wave velocities:
 - prototype: $v_{s1} = 100$ m/s, $v_{s2} = 400$ m/s
 - model: $v_{s1} = 40$ m/s, $v_{s2} = 160$ m/s
- Pile scaling:
 - prototype: $d = 600$
 - model: $d = 20$ mm, $t = 0.146$ mm – not available commercially !
 - $t = 0.710$ mm – compromise !



Scaling Facts

- Seismic input scaling:
When working with scaled models and dynamic tests, one issue is represented by the need for having the seismic excitation giving the right energy input at the right frequency.

Then, since the scaled model will have different natural frequencies from the prototype, the time scale of the input accelerogram must be varied.

This normally implies the need for operating with shorter time-scale hence leading to higher frequency content (here the limitation on the maximum frequency of the shake table comes into play)

- scaling factor for input freq: $S_f = S_L^{0.75} = 15.25$
- scaling an earthquake with dominant freq. of 4 Hz will bring the dominant freq. beyond 60 Hz – **problem ! $S_f = 12$ – compromise !**

Clearly these approximations involve variation of the response, but the obtained one was estimated to be “good enough” to represent the prototype behaviour.



Building the Physical Model

The concrete pile was modelled using an aluminium tube with a number of strain-gauges to capture its flexural behaviour.



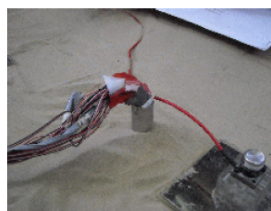
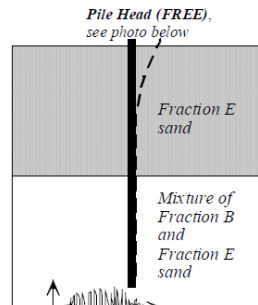
This picture shows the base layer being pluviated in the shear box, the line drawn on the side of the box shows the location of the layers interface.



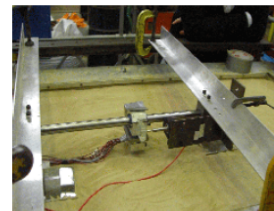
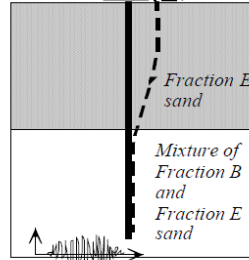
Physical Modelling of Pile Head Conditions

Free and fixed pile end conditions were emulated.

Actually, in the fixed end condition, just the rotation was fixed, so the pile head was still free to translate.



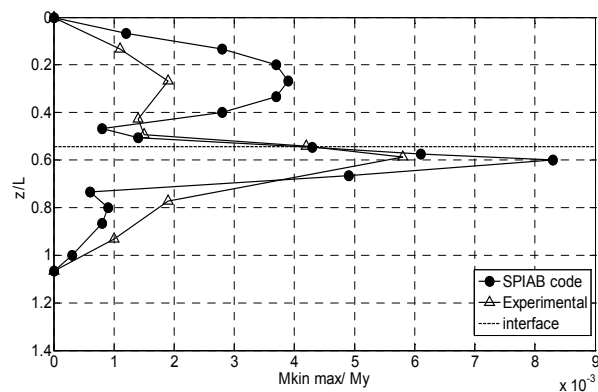
Pile Head (No rotation but free to translate) - see photo below



Pile Bending Response: Experimental vs. SPIAB Code Predictions

Here is reported one of the experiment response in which it is clear that the maximum bending response of the pile is located at the interface between the two soil layer.

The picture also shows the comparison between the experimental data and the numerical results of a FEM model analysis.



Note: - Free head pile, Input: Sturmo A-000,
- SPIAB is a linear elastic analysis code

Effect of scaling in shake table test: summary

Using a scaled model (scaling factor λ) for shake table test implies:

- Lengths, areas and volumes are scaled respectively as λ , λ^2 and λ^3 ;
- Additional masses are required to obtain unscaled stresses acting on the scaled model: these masses influence the gravity loads and the seismic induced inertia forces;
- Additional loads on the prototype (such as those representing the service loads) will be scaled and applied to the model;
- The acceleration time history has to be scaled in time by the square root of the scaling factor $\lambda^{1/2}$;
- The model structural periods and frequencies are scaled respectively as $\lambda^{1/2}$ and $\lambda^{-1/2}$ (i.e. the model periods are shorter than the prototype ones, the scaled frequencies are higher than the prototype ones);
- Measured displacements are scaled by λ ;
- Measured velocity are scaled by $\lambda^{1/2}$;
- Measured acceleration are NOT affected by the scaling.



References

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